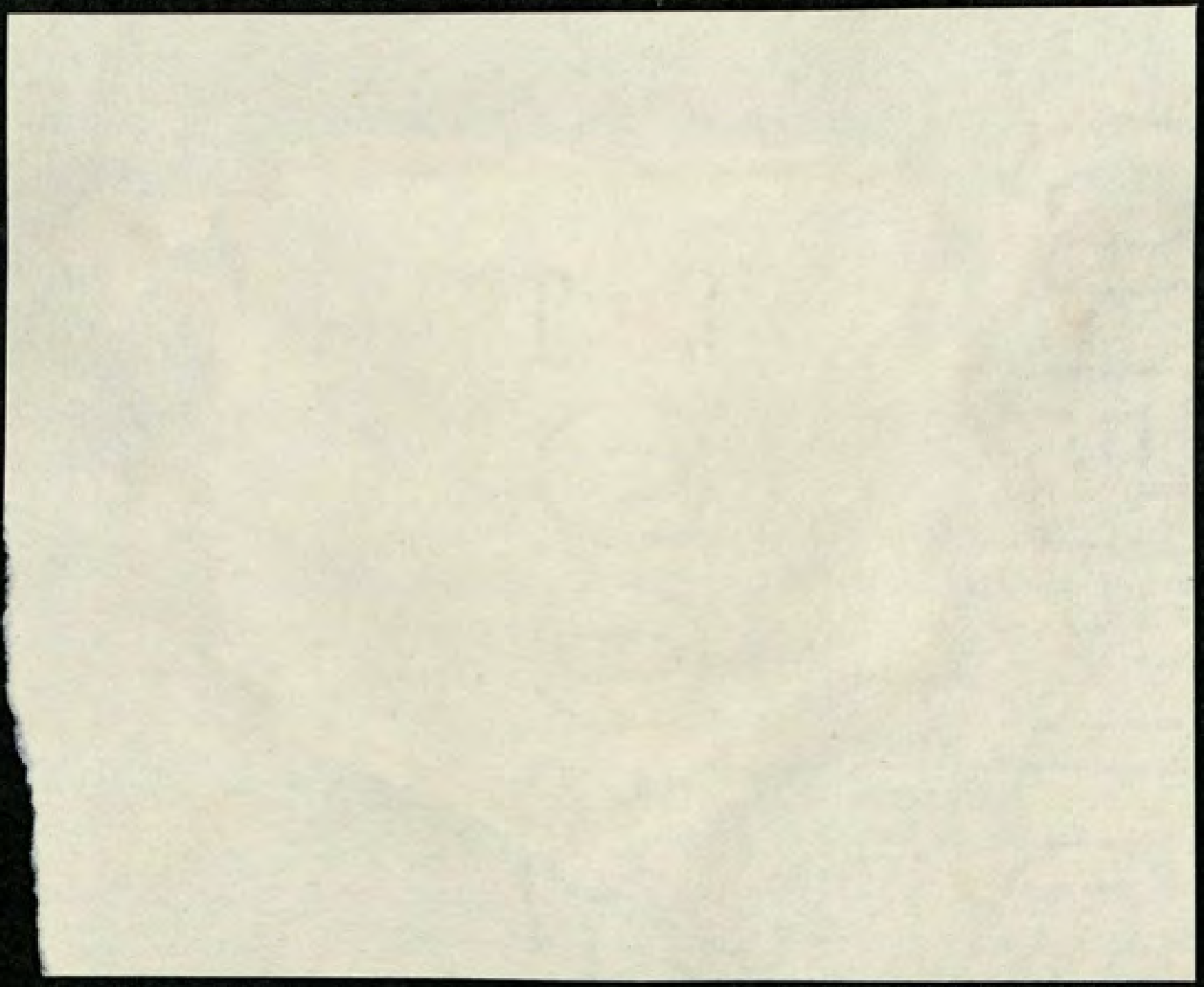
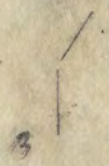
Problems.

1. To erect a perpendicular to a line from a given point in it.

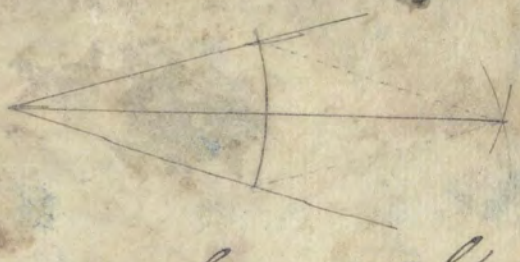
2. To do the same of the point [illegible] the line.



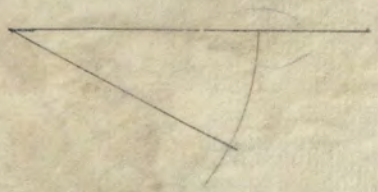
3. To bisect a given line



4. To bisect a given angle.



5. To make an angle equal to a given angle.



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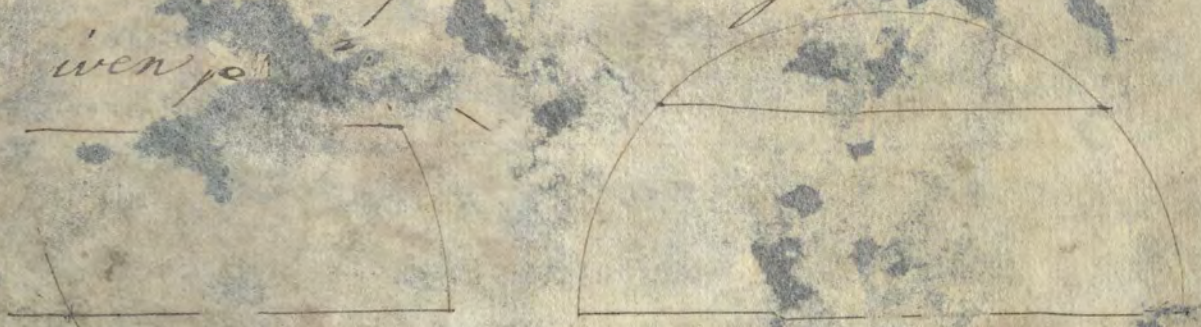
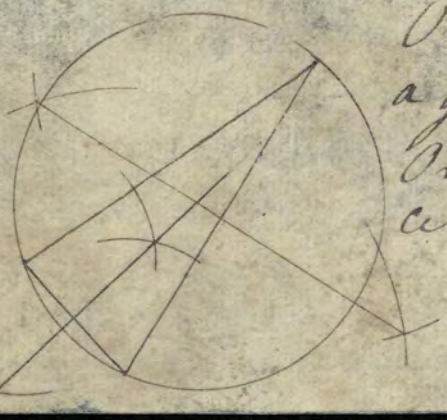
6. To draw a parallel to a given line thro' a given point

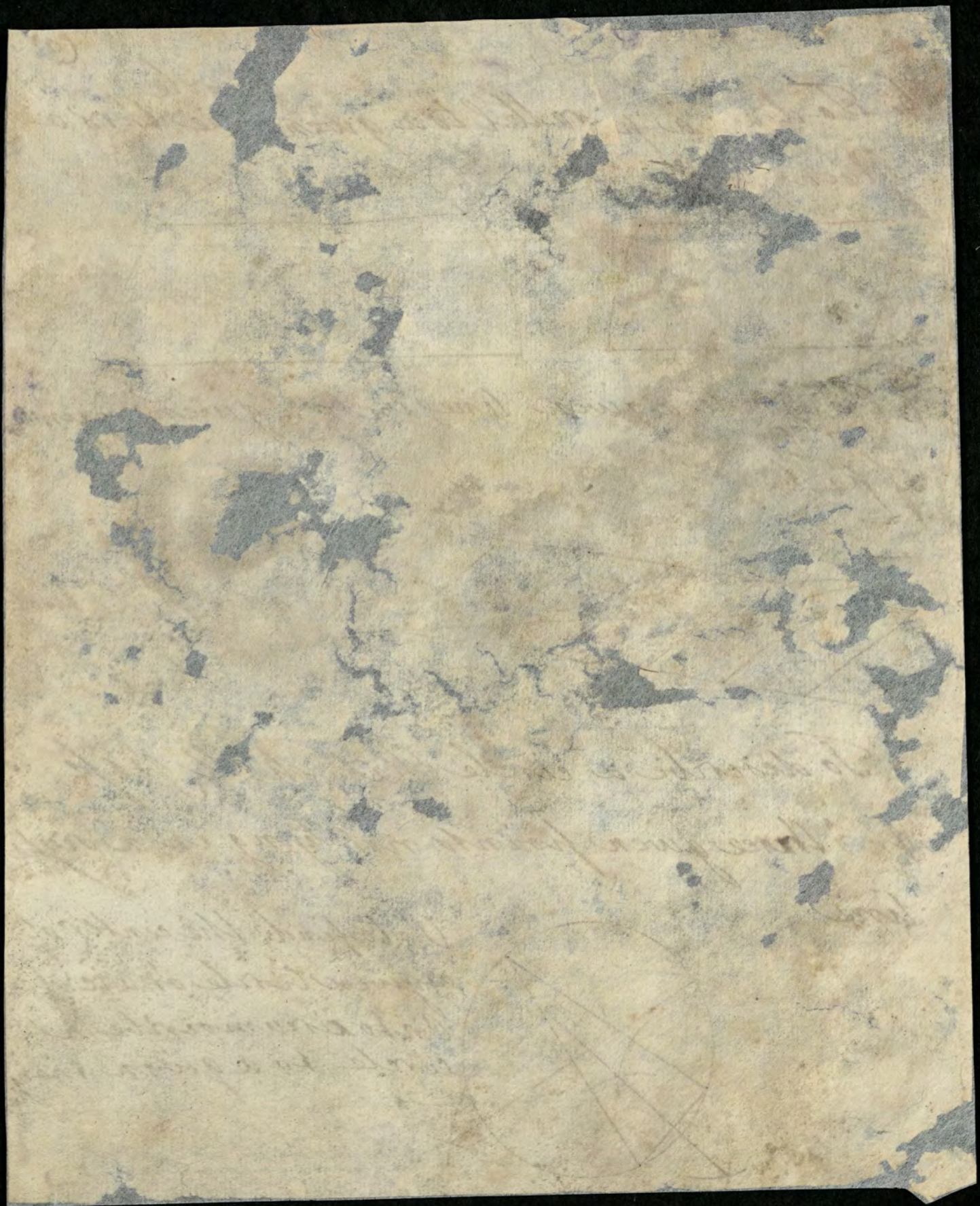


7. To divide a given line into a given number of parts

8. To describe a circle that shall pass thro' three given points not lying in a right line:



Or, to find the center of a given circle or arc:
Or, to circumscribe a circle to a given triangle.

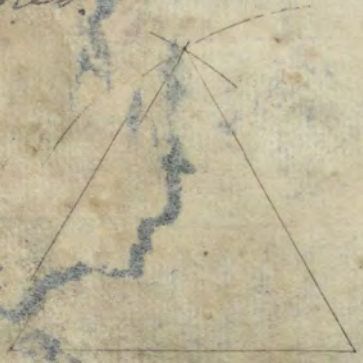


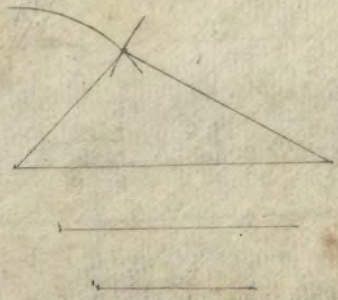
(1828)

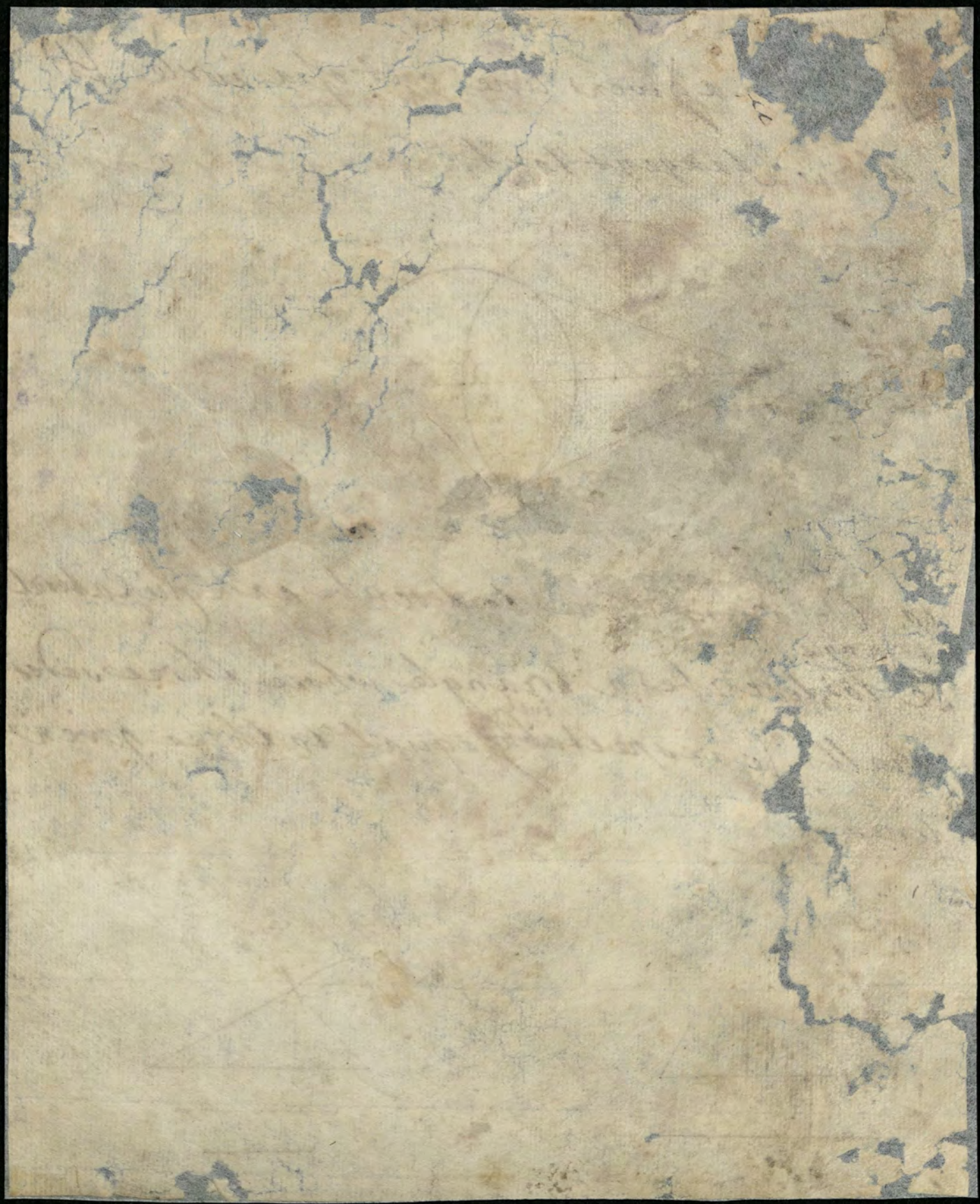
9. [illegible] a given line out of a circle to draw a tangent to it.

10. On a given line to describe an equilateral triangle.

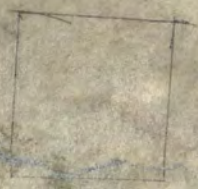
[illegible] to describe a triangle whose three sides shall be respectively equal to three given lines.

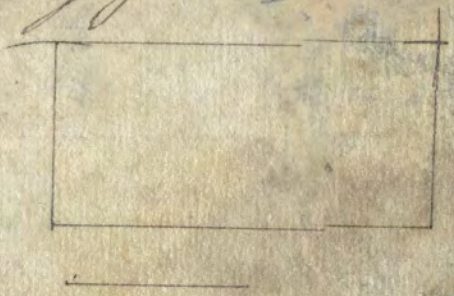




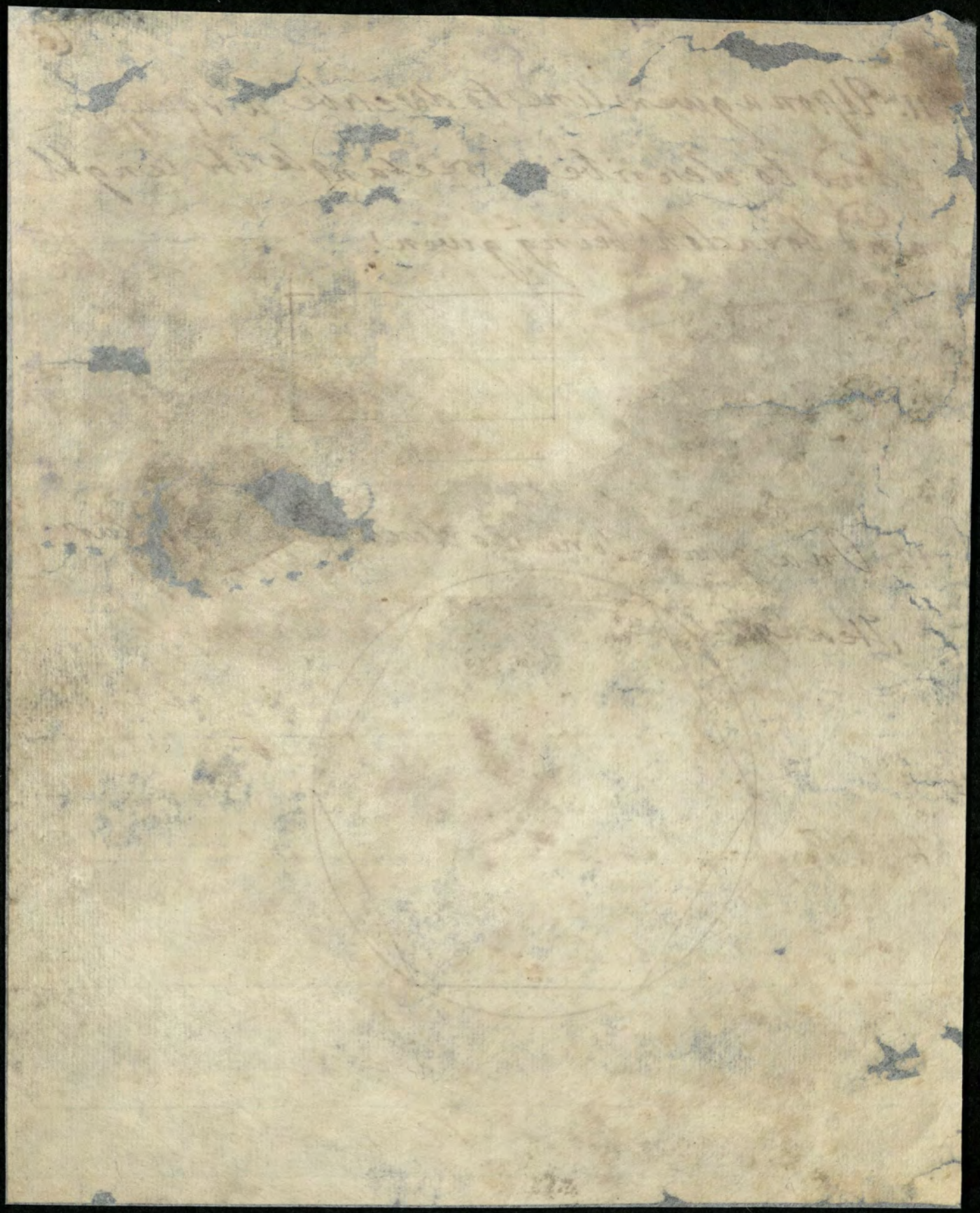


11. Upon a given line to describe a square.
And to describe a rectangle its length
and breadth being given.



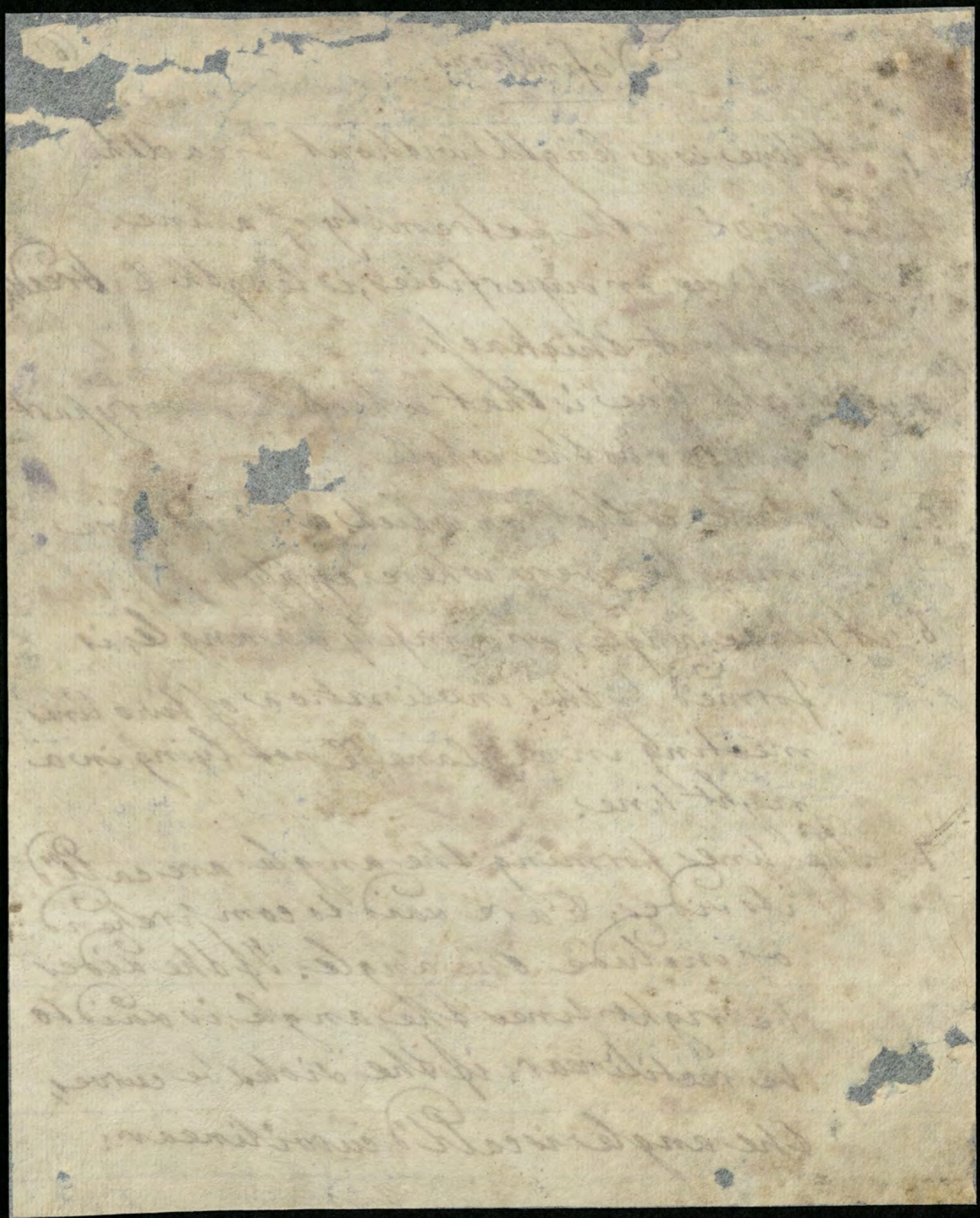


12. On a given line to describe a regular
Hexagon.



Definitions

1. A line is a length without breadth.
2. A point is the extremity of a line.
3. A surface, or superficies, is length & breadth without thickness.
4. A right line is that which has every part similar to the whole.
5. A plane is that on which a right line may be every where applyd
6. A plane angle, or simply an angle, is formed by the inclination of two lines meeting in a plane, & not lying in a right line.
7. The lines forming the angle are calld its sides, & are said to comprehend or include the angle. If the sides be right lines the angle is said to be rectilinear: if the sides be curve, the angle is calld curvilinear:



& if one be streight & the other curve the angle is call'd mixtilinear.

8. If one right line meet another so as to form two equal adjacent angles, each of these is call'd a right angle, & the line forming them is call'd a perpendicular, & some times a Normal.

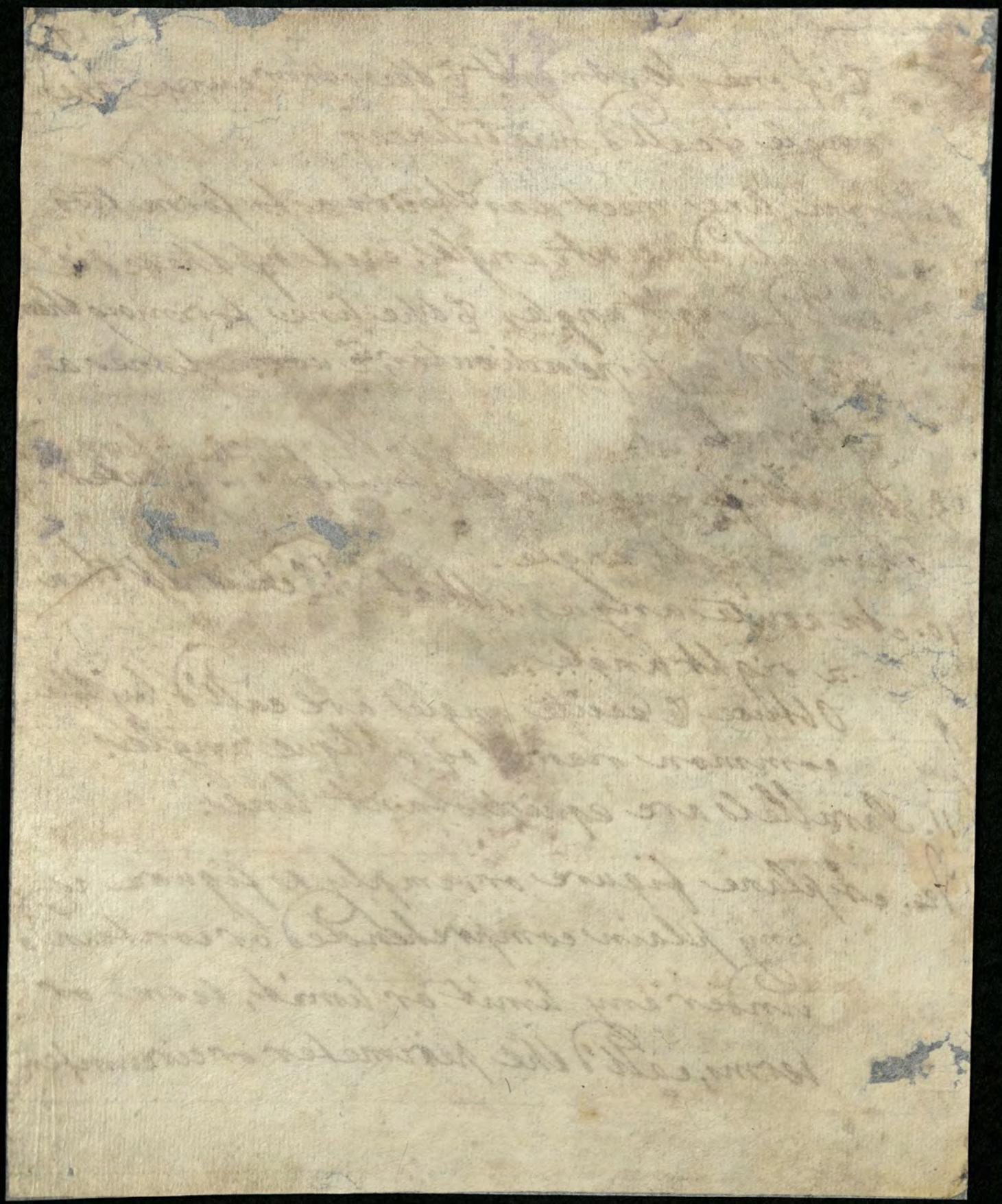
9. An obtuse angle is that which is greater than a right angle.

10. An acute angle is that which is less than a right angle.

Obtuse & acute angles are call'd by the common name of oblique angles.

11. Parallels are equidistant lines.

12. A plane figure or simply a figure is any plain comprehended or contain'd under any limit or limits, term or terms, call'd the perimeter or circumference.



13. A circle is a plane figure terminated by one line, called its Periphery or circumference, every where equally distant from a point within the circle, call'd its center.

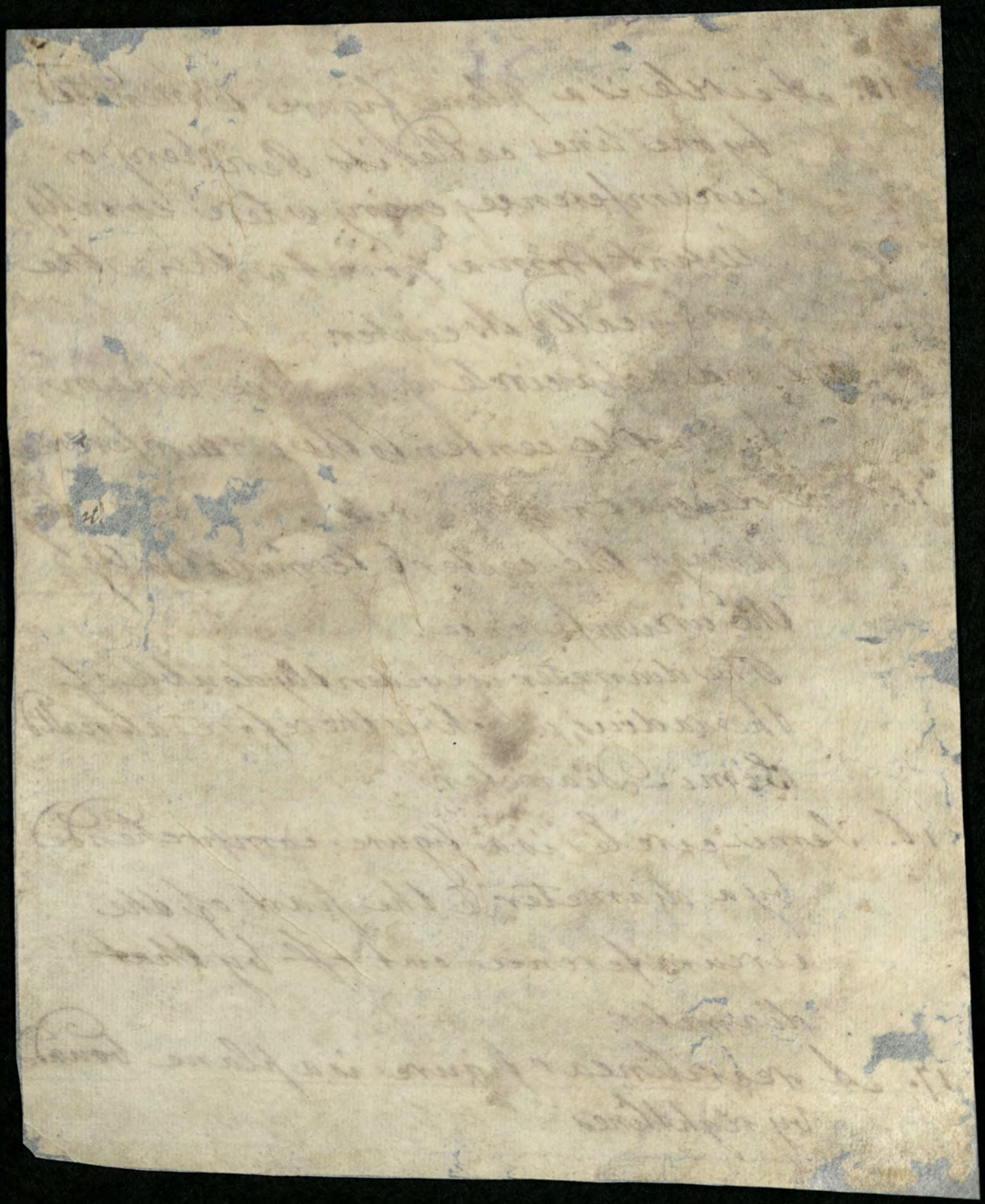
14. The radius of a circle is any line drawn from the center to the circumference.

15. The diameter of a circle is a line passing through the center & terminated by the circumference.

The diameter is evidently double of the radius, which is therefore also call'd Semi-Diameter.

16. Semi-circle is a figure comprehended by a diameter & the part of the circumference cut off by that diameter.

17. A rectilinear figure is a plane bounded by right lines



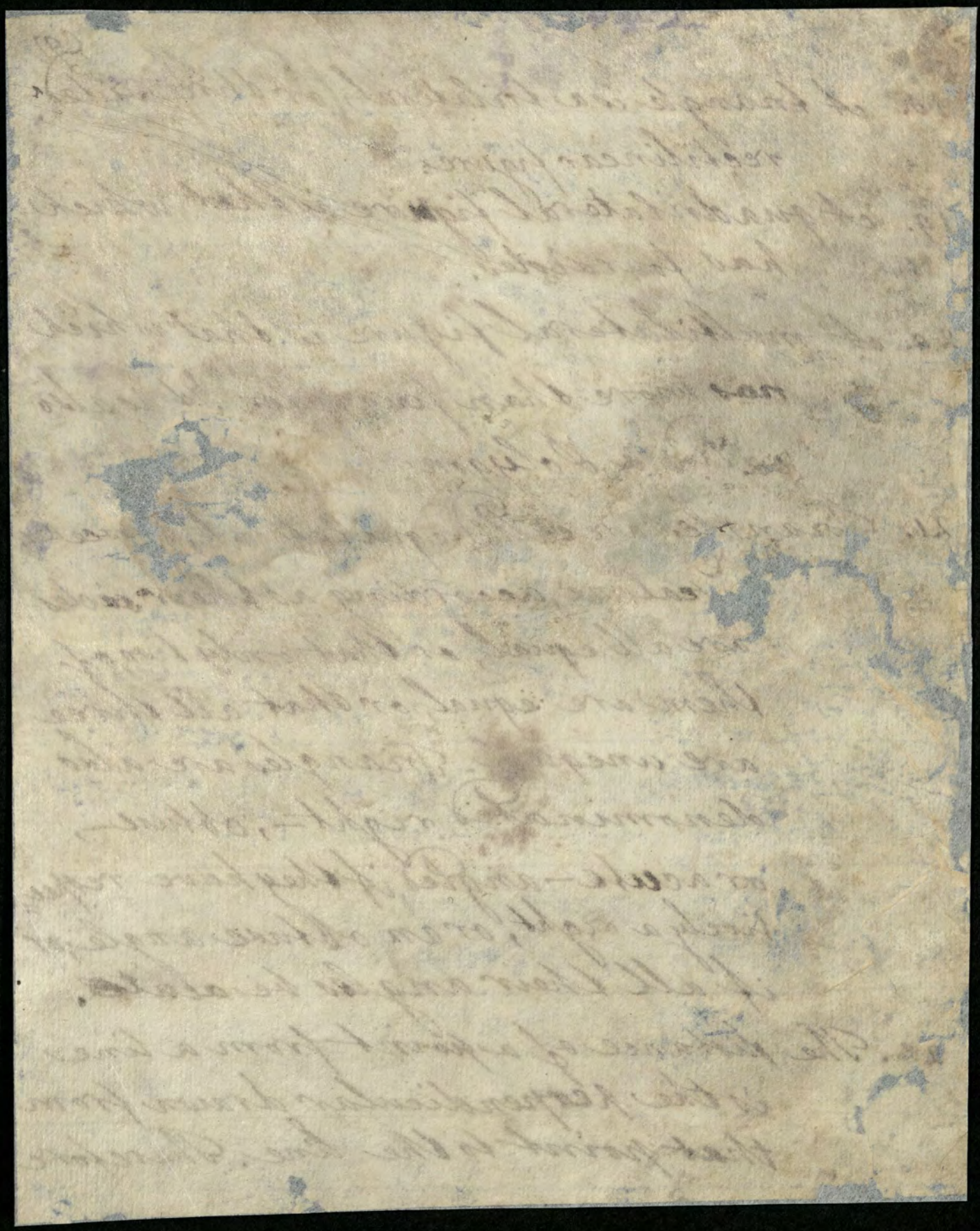
18. A triangle is a trilateral, or three sided rectilinear figure.

19. A quadrilateral figure is that which has four sides.

20. A multilateral figure is that which has more than four sides. It is also call'd a Polygon.

21. Triangles are call'd equilateral, isosceles, scalene, according as their sides are all equal, or that only two of them are equal, or that all three are unequal. Triangles are also denominated right-, obtuse-, or acute-angled, if they have respectively a right, or an obtuse angle, or if all their angles be acute.

22. The distance of a point from a line is the perpendicular drawn from that point to the line. Therefore



if a line be parallel to another all the perpendiculars drawn from the first line to the second must be equal.

23. A Parallelogram is a quadrilateral figure the opposite sides of which are parallel.

24. A square is an equilateral rectangular parallelogram.

25. A rectangle or oblong is a right-angled parallelogram not equilateral.

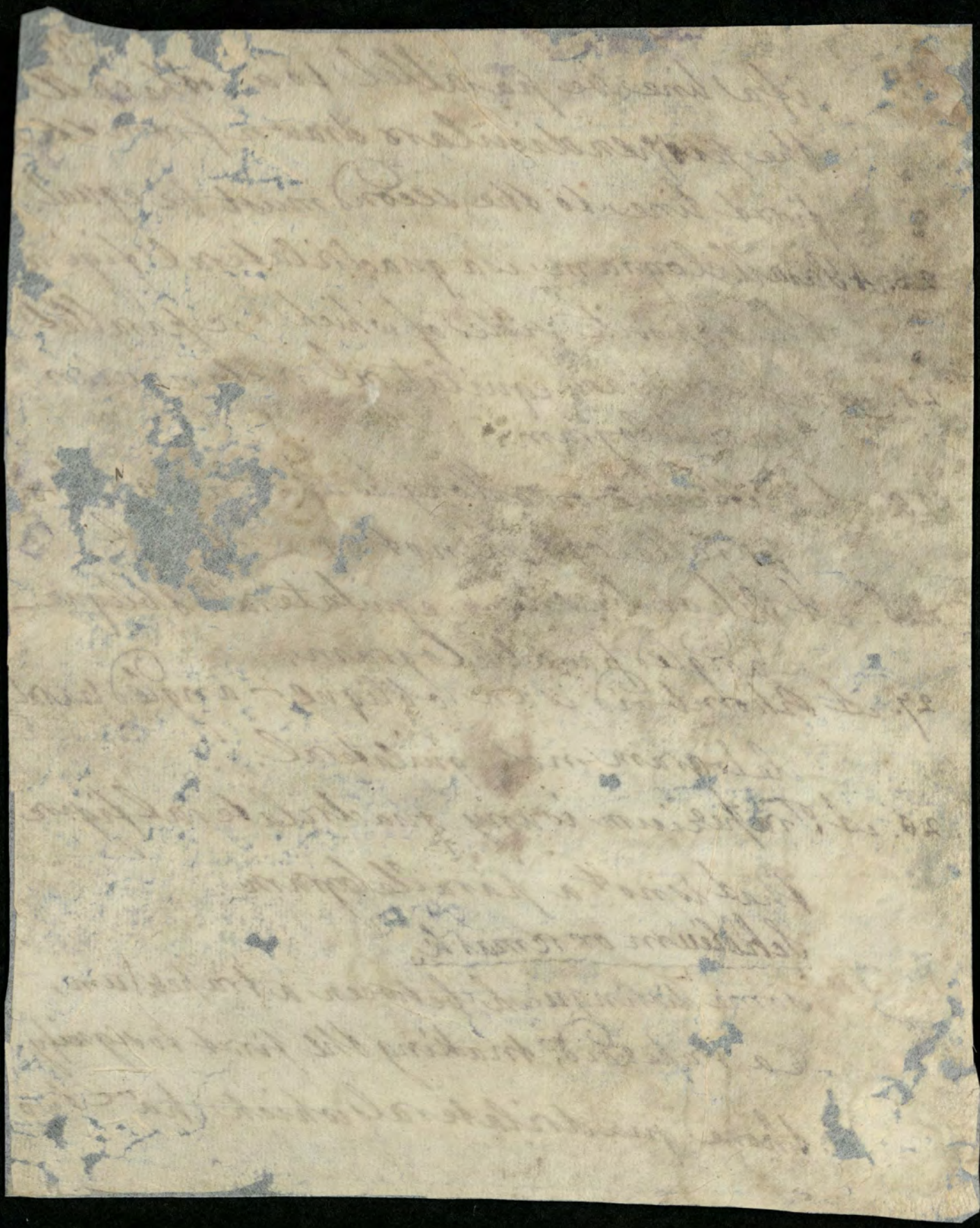
26. A Rhombus is an equilateral oblique-angled parallelogram.

27. A Rhomboid is an oblique-angled parallelogram not equilateral.

28. A Trapezium is any quadrilateral figure that is not a parallelogram.

Scholium or remark.

Some distinguish between a trapezium, & a trapezoid, making the first to signify those quadrilaterals which have two



sides parallel; & the second, those that have no parallel sides.

29. A diagonal is a right-line drawn through the opposite angles of a quadrilateral. Some call it a diameter.

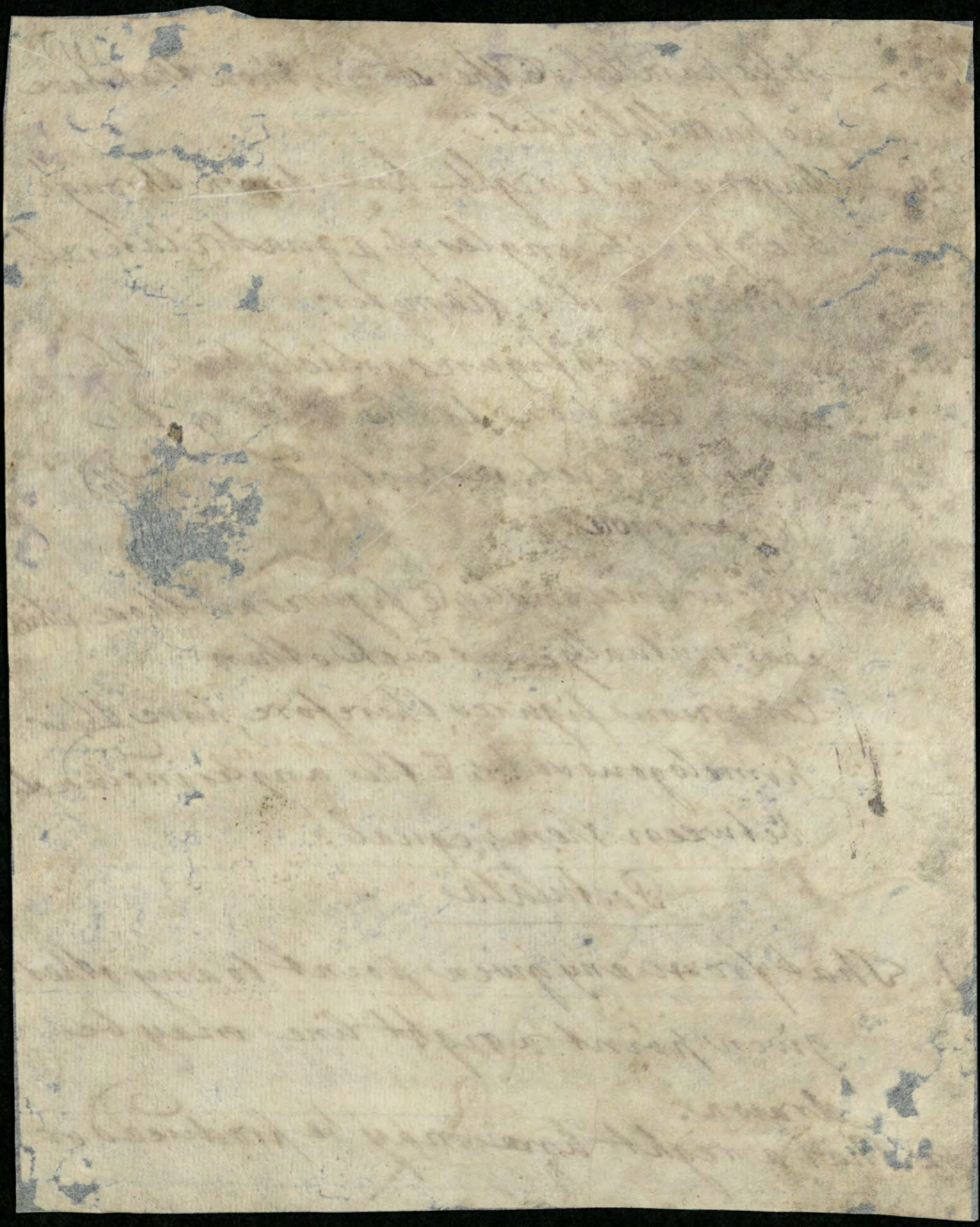
30. The sides of two figures which have the same relation to the rest of the sides in each respectively are called Homologous.

31. Congruous lines, angles, & figures are those which can mutually cover each other. Congruous figures, therefore, have their homologous sides, & the angles included between them, equal.

Postulata

1. That from any given point to any other given point a right line may be drawn.

2. That a right line may be produced or



continued at pleasure.

3. That from any center, with any radius, the circumference of a circle may be discribed.

4. That a right line may be drawn from any point equal to a given right line.

5. That two unequal right lines being given a part may be taken from the longer equal to the shorter.

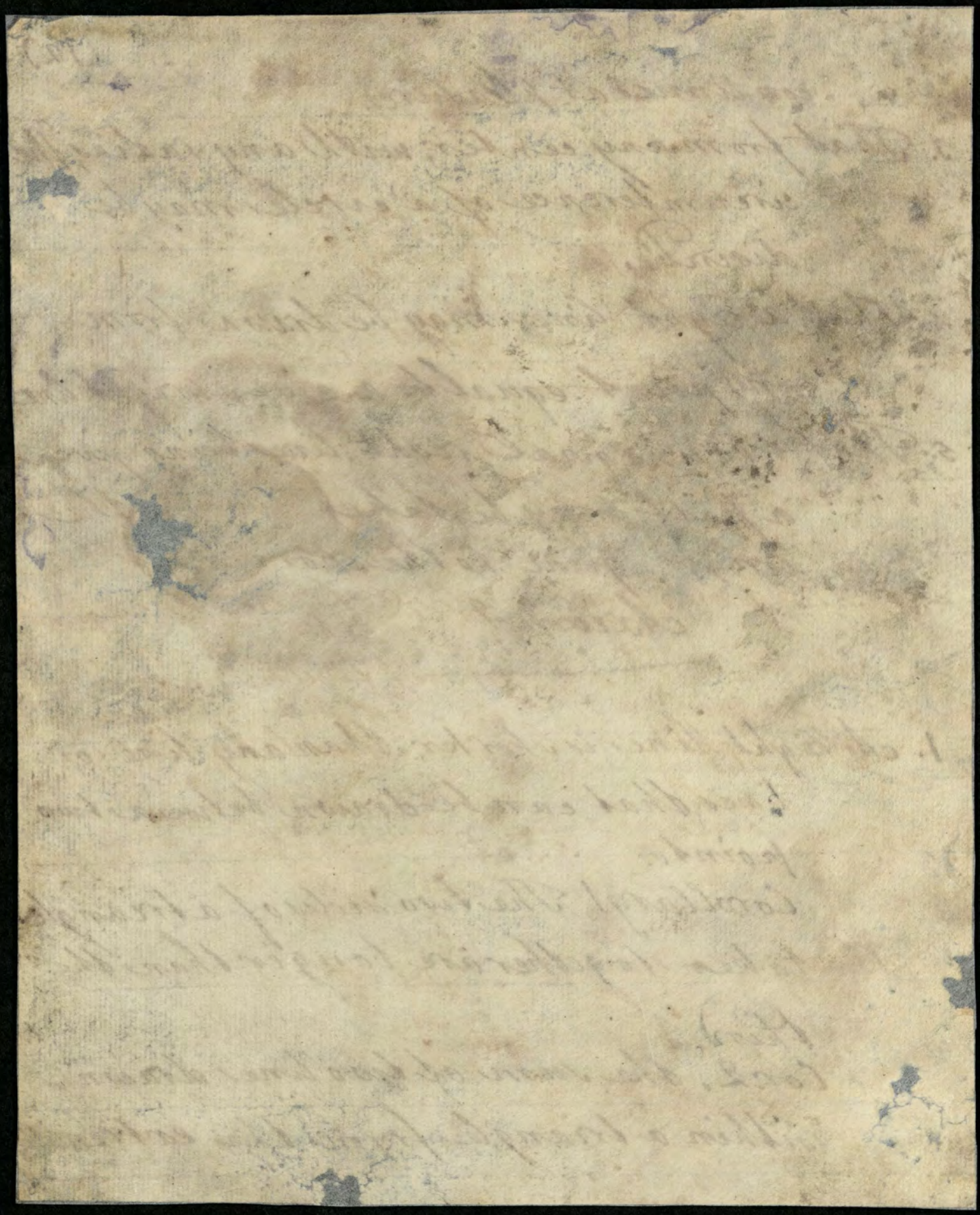
Axioms.

---

1. A right line is shorter than any line, or lines, that can be drawn between two points.

Corollary 1. The two sides of a triangle taken together are longer than the third.

Cor. 2. The sum of two lines drawn within a triangle from the extremity



of its base is less than the sum of its sides.

2. Two right lines cannot have more than one common point.

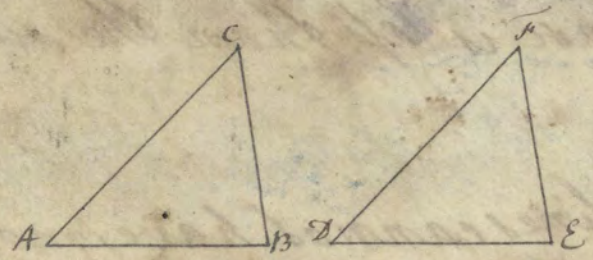
Cor. Therefore two right lines cannot comprehend a space, nor have a common segment.

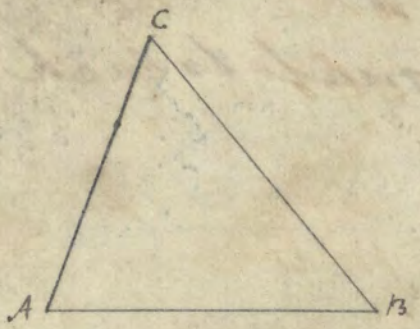
3. The parallel to a right line is a right line.

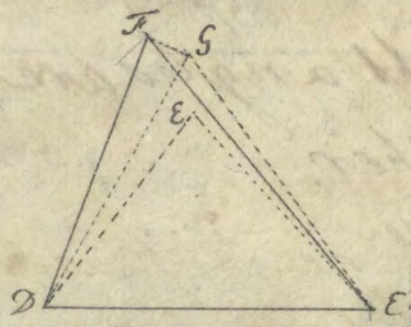
4. Congruous lines, angles, & figures, are equal.

5. Equal lines & angles (not figures) are congruous.

6. All right angles are equal to each other.







Theorems.

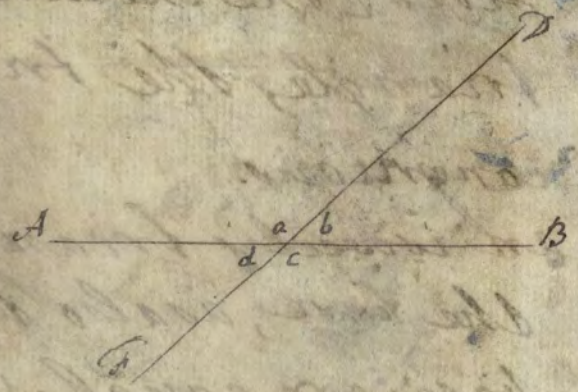
I. If a triangle have two sides and the included angle equal respectively to the two sides & the included angle of another triangle, the triangles will be congruous.

II. In an isosceles triangle the angles at the base, as also those under the base, are equal: & conversely

III. If the three sides of a triangle be respectively equal to the three sides of another, the triangles will be congruous.

Cor. Hence appears the method
1. Of bisecting an angle. (Prob. 4)
2. Of bisecting a line. (Prob. 3)
3. Of raising a perpendicular. (Prob. 1.)
4. Of letting fall a perpendicular. (Prob. 2)

5.



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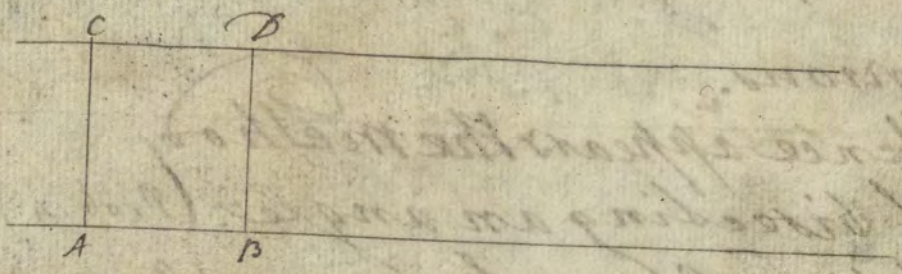


(1839)

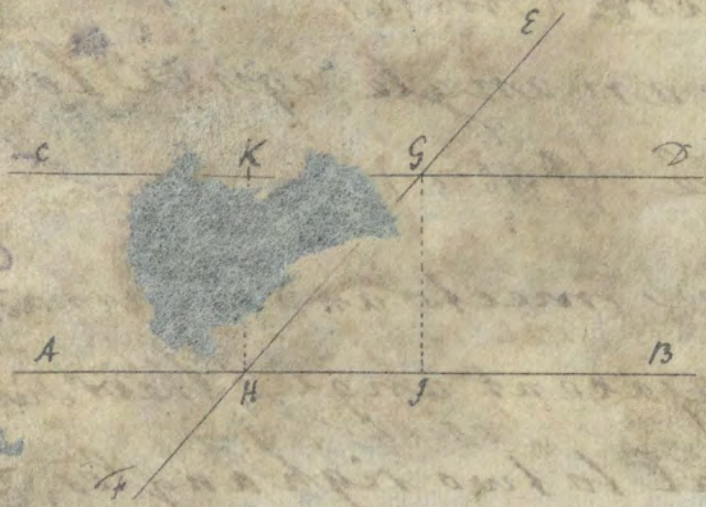
5. Of describing a triangle the three sides of which shall be equal, respectively, to three given lines; if any two of them be greater than the third. (Prob 10.)

6. Of making an angle equal to a given angle. (Prob. 5.)

IV. If one right line meets another so as to form two adjacent angles, their sum will be equal to two right angles; & if the lines intersect each other, the opposite angles will be equal: & conversely.

Lemma! A right line equidistant in two points from another right line is equidistant in all points.

Cor. Hence may be deduced a method of drawing a parallel to a given



right line from a given point.

Lemma 2. The perpendicular to one parallel is perpendicular to the other.

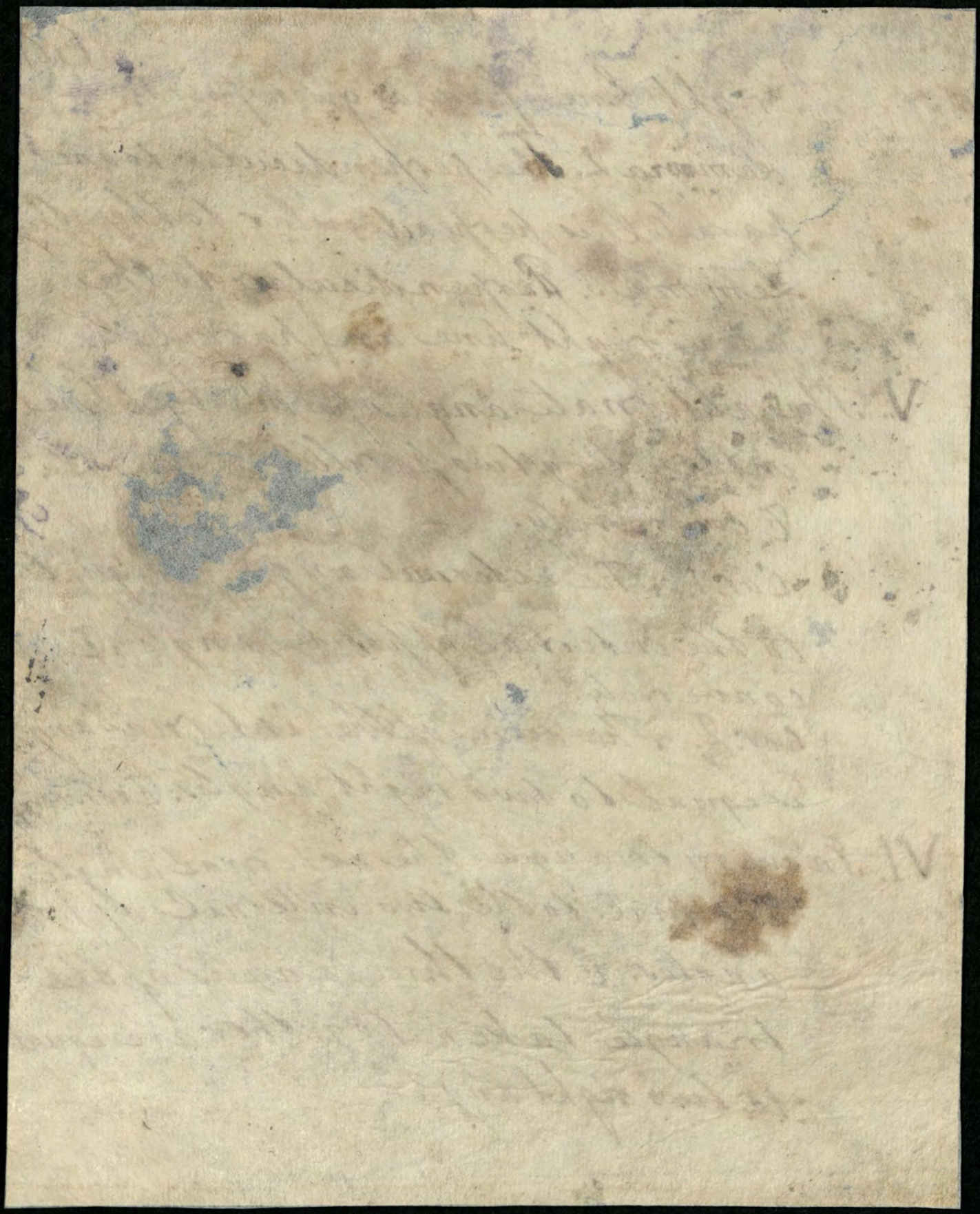
Lemma 3. Perpendiculars to the same right line are parallel.

V. The alternate angles form'd by a line intersecting two parallels are equal: & conversely.

Cor. 1. The external angle equal to the internal opposite angle: & conversely.

Cor. 2. The sum of the internal angles is equal to two right angles: & converse.

VI. In every triangle the external angle is equal to the two internal opposite angles. & the three angles of the triangle taken together are equal to two right angles.

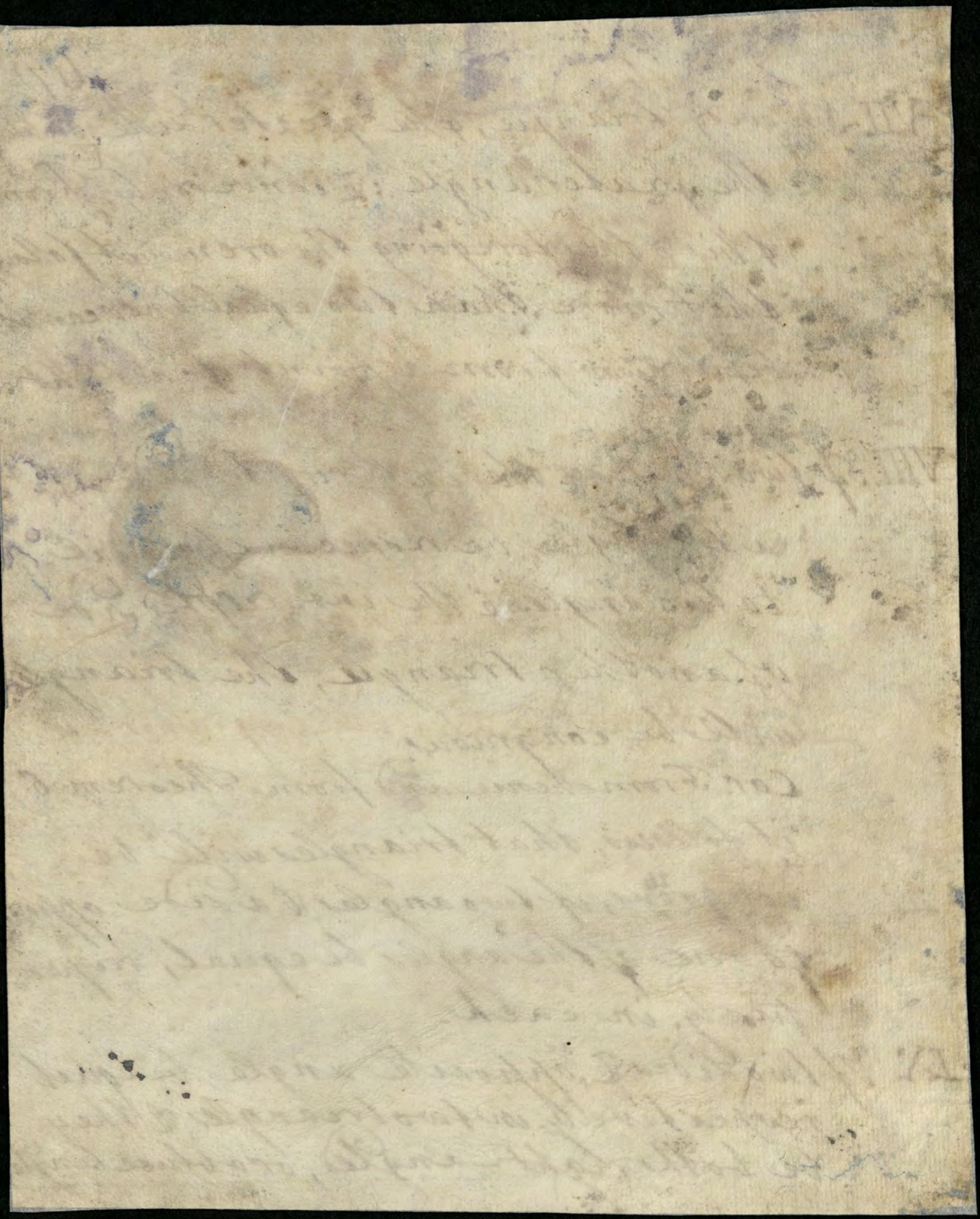


VII. In every triangle, the greater side subtends the greater angle: & conversely. [Cor. From this & the foregoing theorem it follows, that more than two equal lines cannot be drawn from a point to a right line.

VIII. If two angles & the intercepted side of a triangle be respectively equal to two angles & the intercepted side of another triangle, the triangles will be congruous.

Cor. From hence and from Theorem 6 it follows, that triangles will be congruous, if two angles & a side opposite to one of the angles, be equal, respectively, in each.

IX. If two sides & an opposite angle be equal respectively in two triangles, & they be both right-angled, or obtuse angled



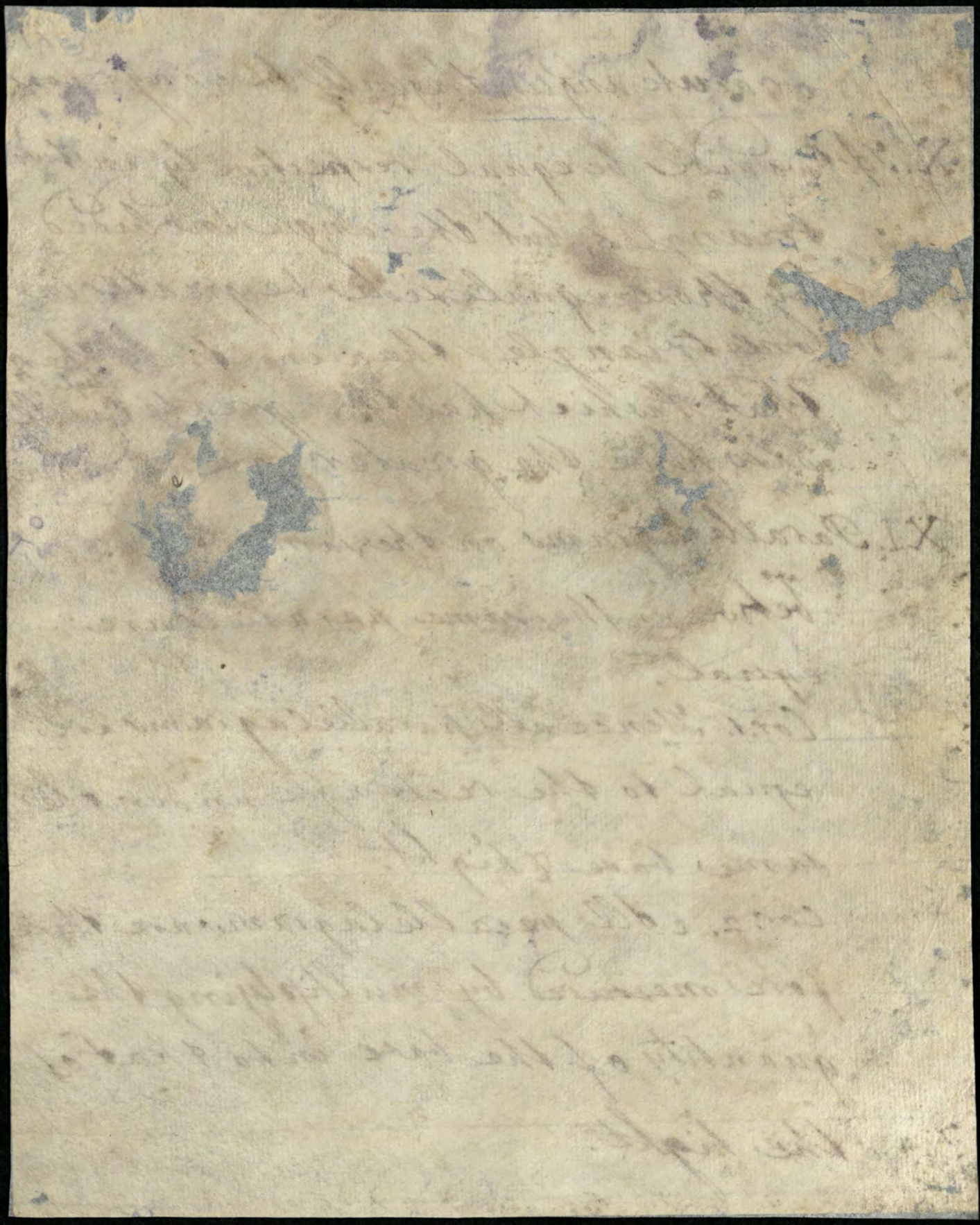
or acute angles, they will be congruous.

X. If two sides be equal respectively in two triangles, but the angle included by those equal sides be greater in one triangle than in the other, that in which has the greater angle will also have the greater side & conversely.

XI. Parallelograms on the same base & between the same parallels are equal.

Cor. 1. Hence all parallelograms are equal to the rectangle under the same base & height.

Cor. 2. All parallelograms are therefore measured by multiplying the quantity of the base into that of the height.



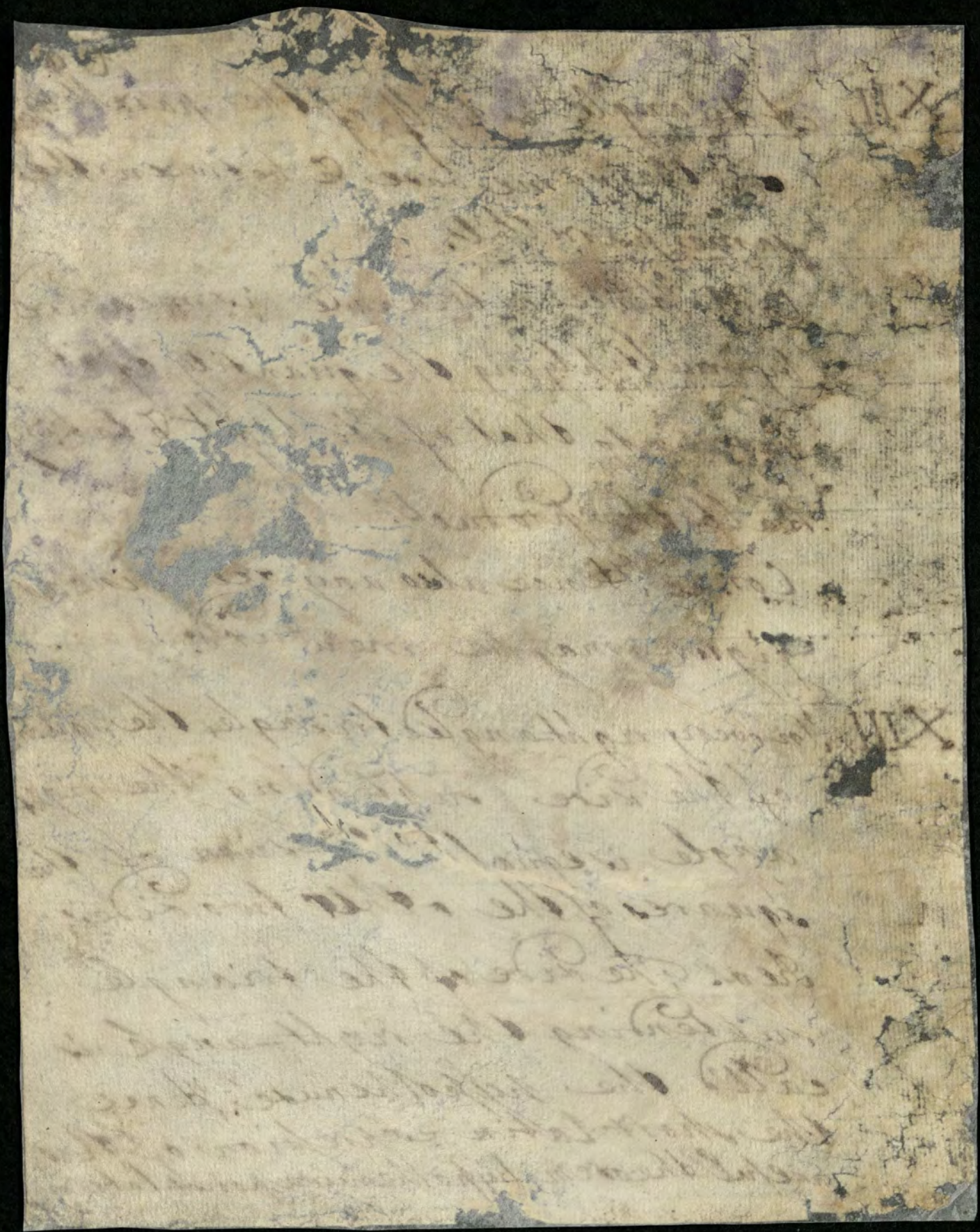
XII. A triangle is half of the parallelogram on the same base & between the same parallels.

Cor. 1. Hence a triangle is measured by multiplying the quantity of its base into that of its height & taking half the product.

Cor. 2. Hence also any rectilineal figure may be measured.

XIII. In every right-angled triangle the square of the side subtending the right angle is equal to the sum of the squares of the other two sides.

Rem. The side of the triangle subtending the right-angle is call'd the hypothenuse. Hence the short Latin expression of this useful theorem, hypothenusa potest latera.



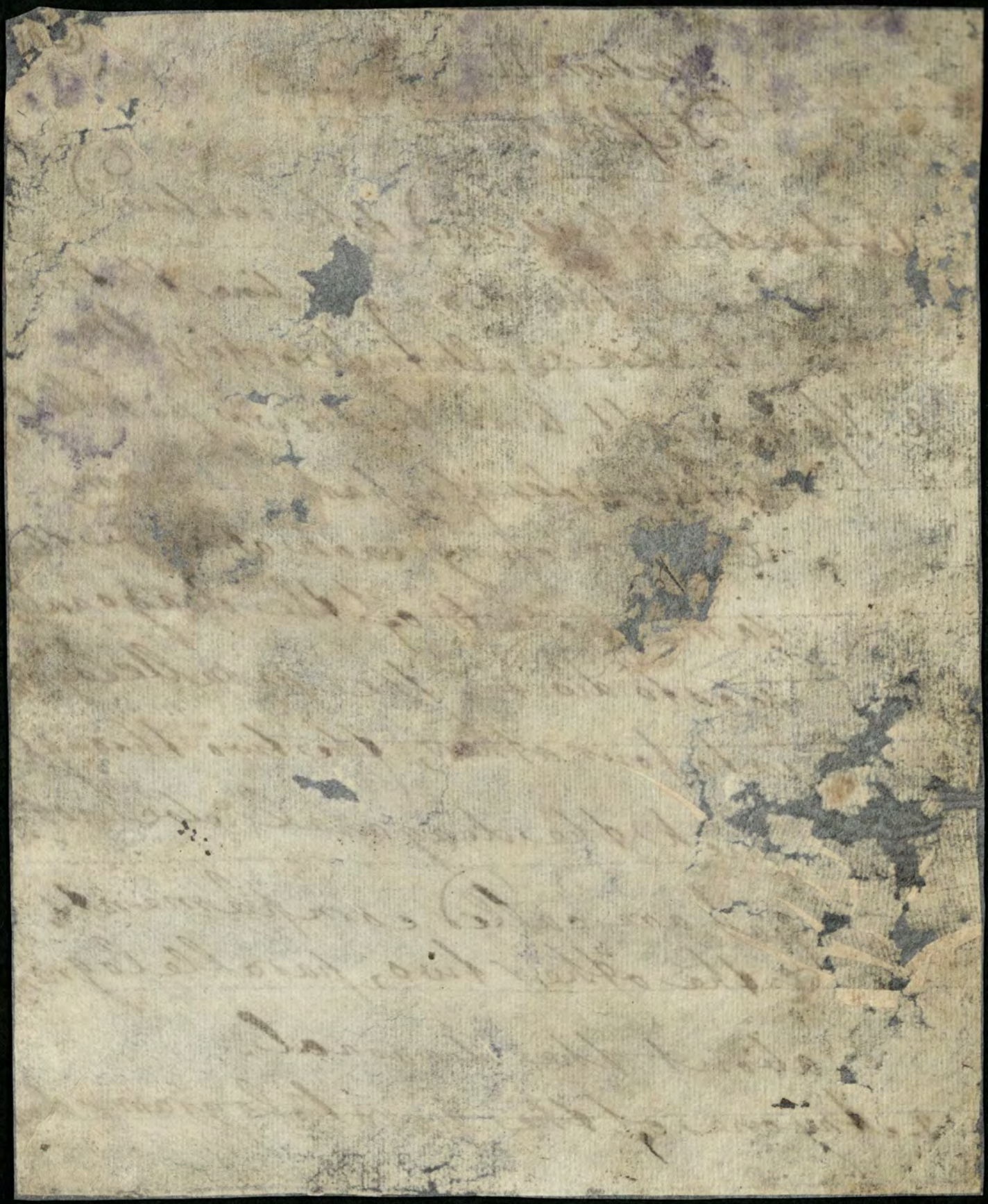
Section II.

Definitions

1. A rectangle is said to be contained under the two right lines that are its base & altitude or height.

2. If two right lines be drawn parallel to the sides of a parallelogram, & meeting each other in the same point of the diagonal, and so to divide the parallelogram into four others, the two through which the diagonal does not pass are called complements, & the other two, parallelograms about the diagonal.

3. Any one of the parallelograms about



(1845)

the diagonal, with the complements is called a Gnomon.

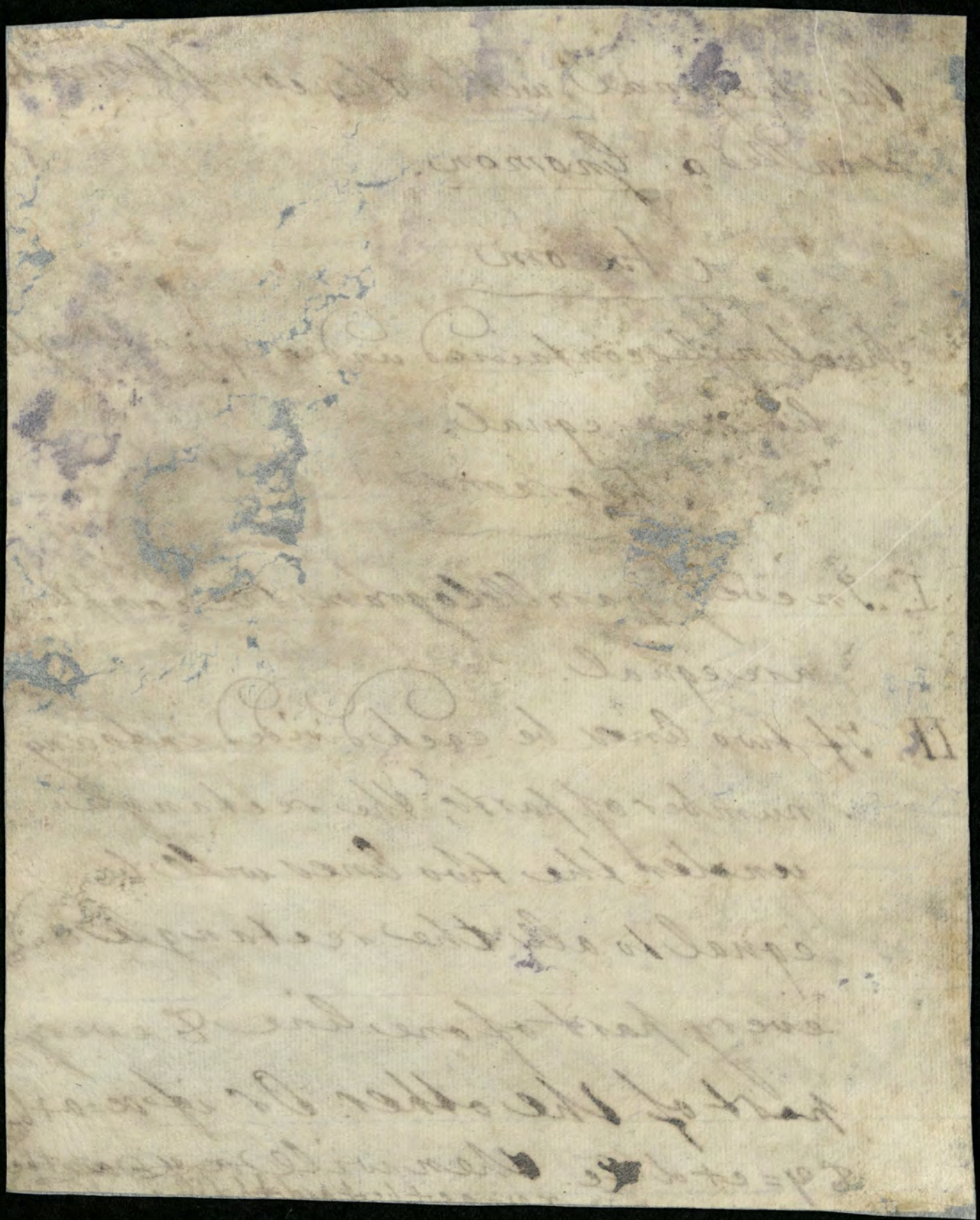
Axiom

Rectangles contained under equal right lines are equal.

Theorems.

I. In every parallelogram the complements are equal.

II. If two lines be each divided into any number of parts, the rectangle under the two lines will be equal to all the rectangles under every part of one line & every part of the other. Or if $x = a + b$ &c & $y = c + d$ &c. then will xy be the $xy = ac + bc + ad + bd$ &c.

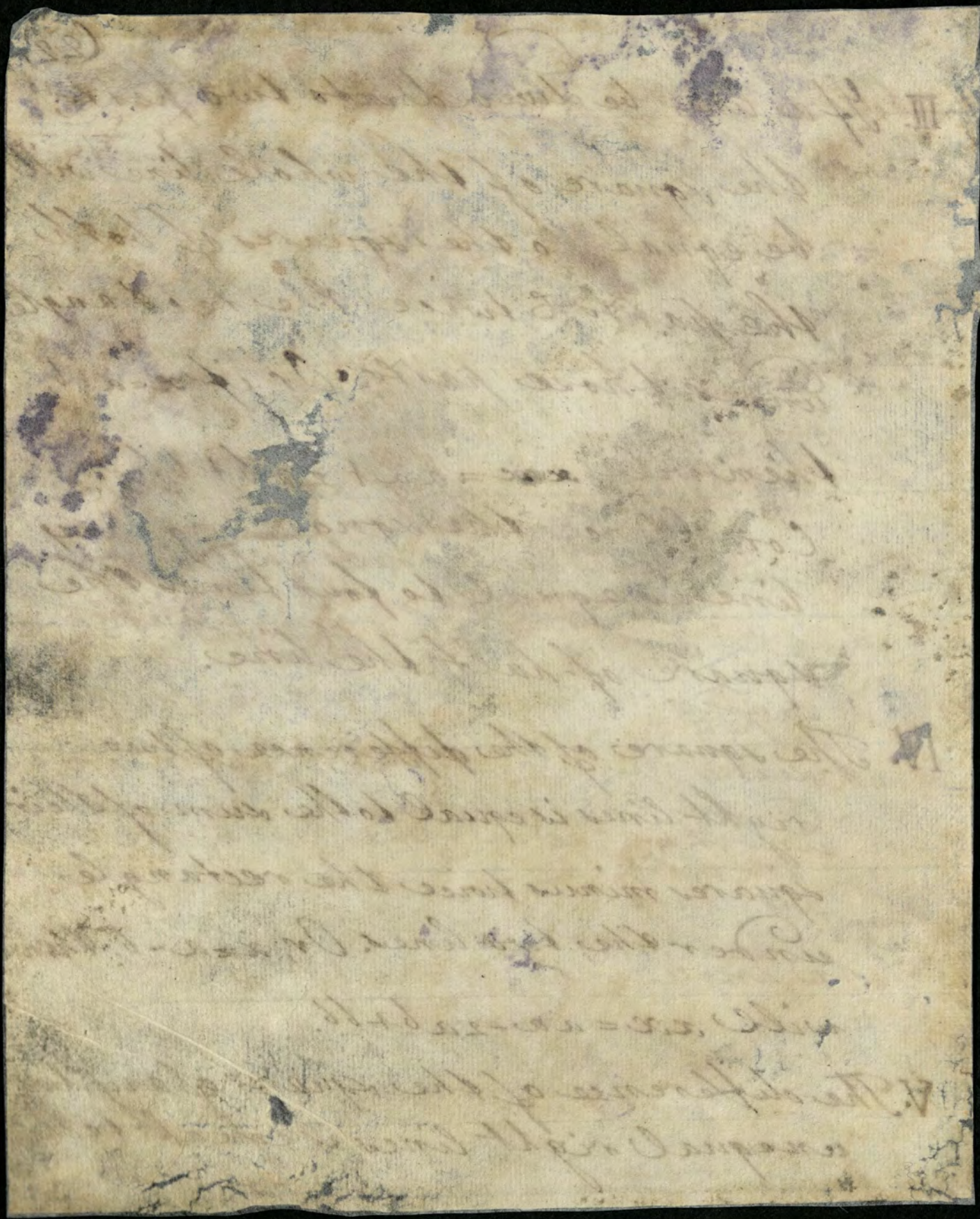


III. If a line be divided into two parts, the square of the whole line will be equal to the squares of both the parts & twice the rectangle under those parts. Or if $x = a + b$, then will $xx = aa + 2ab + bb$.

Cor. Hence the square of any line is equal to four times the square of half the line.

IV. The square of the difference of two right lines is equal to the sum of their squares minus twice the rectangle under the two lines. Or, $x = a - b$, then will $xx = aa - 2ab + bb$.

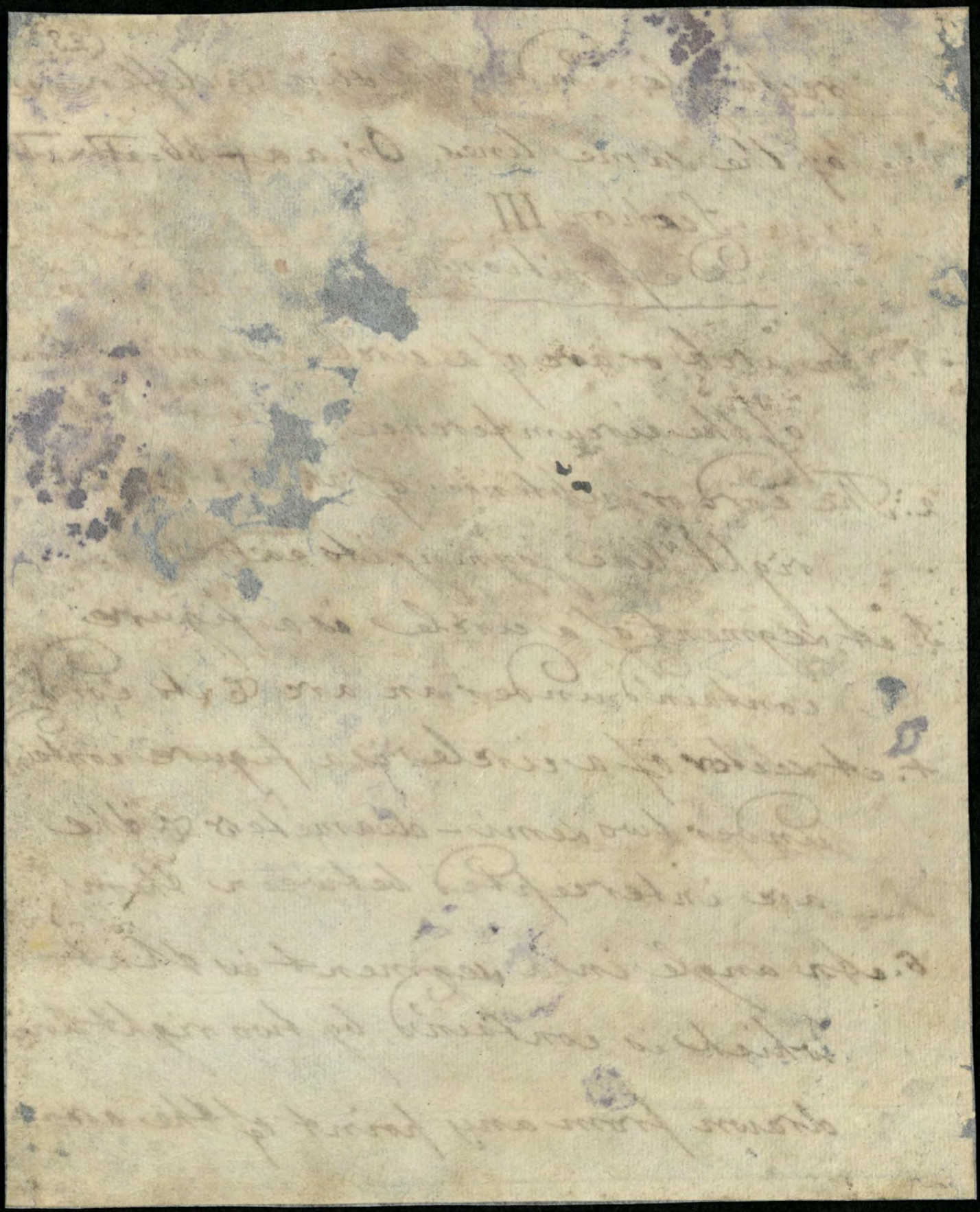
V. The difference of the squares of any two unequal right lines is equal to the



rectangle under the sum & difference
of the same lines. Or; $aa + bb = a+b \times a-b$.

Section III.
Definitions

1. An arch or arc of a circle is any portion of the circumference.
2. The chord or subtense of an arc is the right line joyning its extremes.
3. A segment of a circle is a figure contain'd under an arc & its chord.
4. A sector of a circle is a figure contain'd under two semi-diameters & the arc intercepted between them.
5. An angle in a segment is that which is contain'd by two right lines drawn from any point of the arc



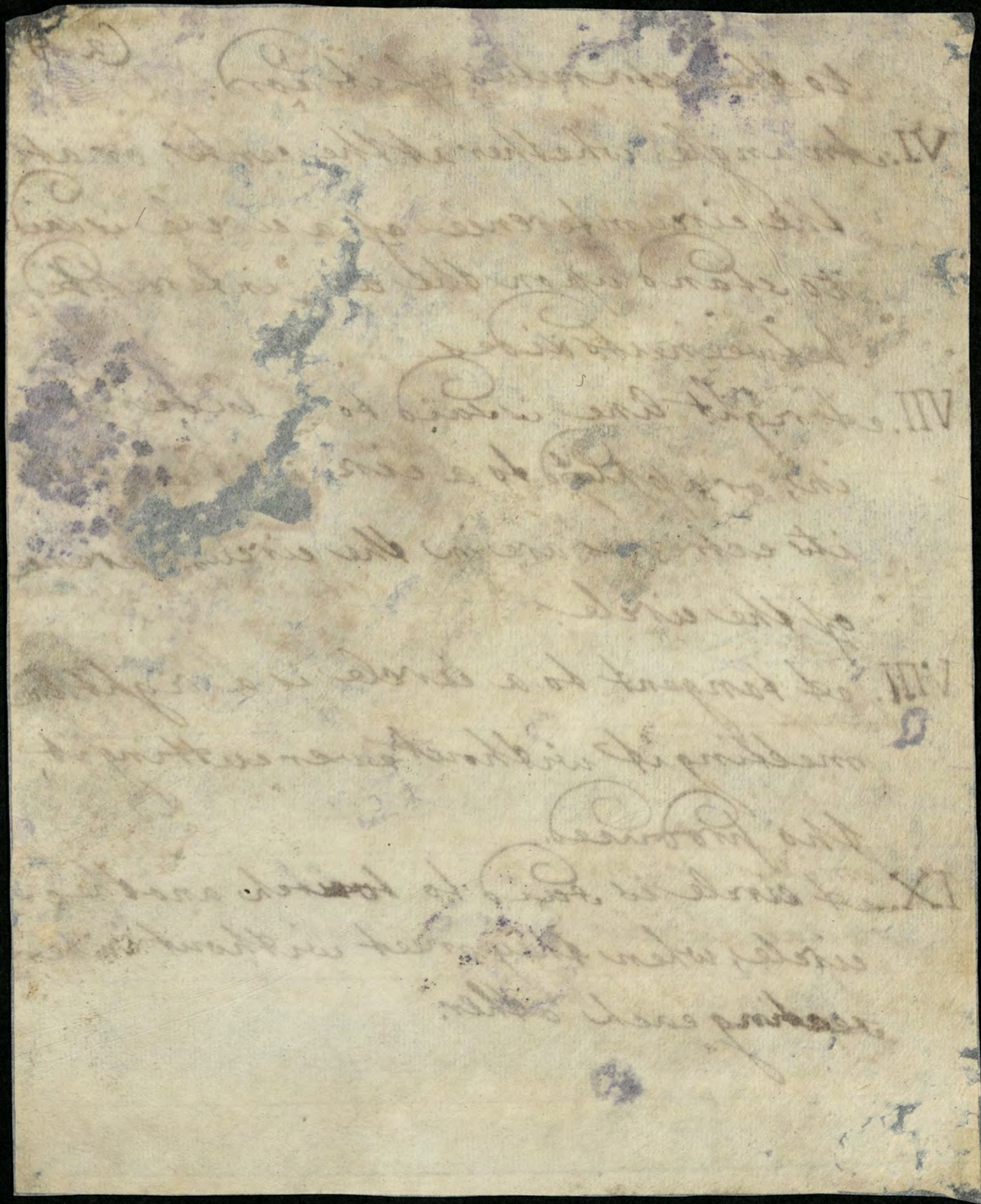
to the extremities of its Chord.

VI. An angle whether at the center, or at the circumference of a circle is said to stand upon the arc intercepted between its sides.

VII. A right line is said to be inscribed in, or applied to a circle when both its extremities are in the circumference of the circle.

VIII. A tangent to a circle is a right line meeting it without ever cutting it, tho' produced.

IX. A circle is said to touch another circle, when they meet without intersecting each other.



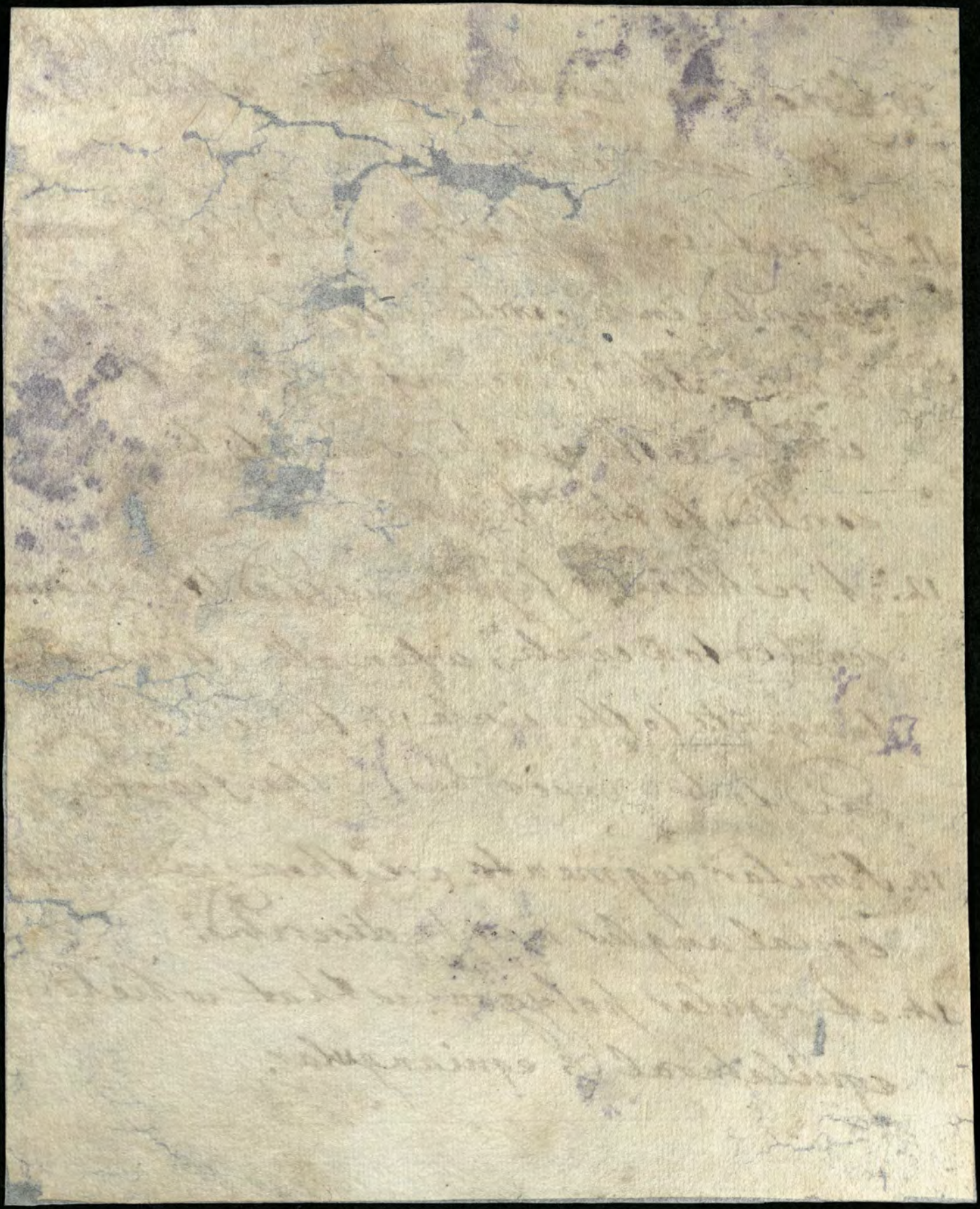
10. Concentric Circles are those which have the same center.

11. A rectilinear figure is said to be inscribed in a circle when all its angles are in the circumference of the circle: & the circle is said to be circumscribed to the figure.

12. A rectilinear figure is said to be circumscribed to a circle, when all its sides are tangents to the circle: & the circle is said to be inscribed in the figure.

13. Similar segments are those in which equal angles may be described.

14. A regular polygon is that which is equilateral & equiangular.



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Axiom

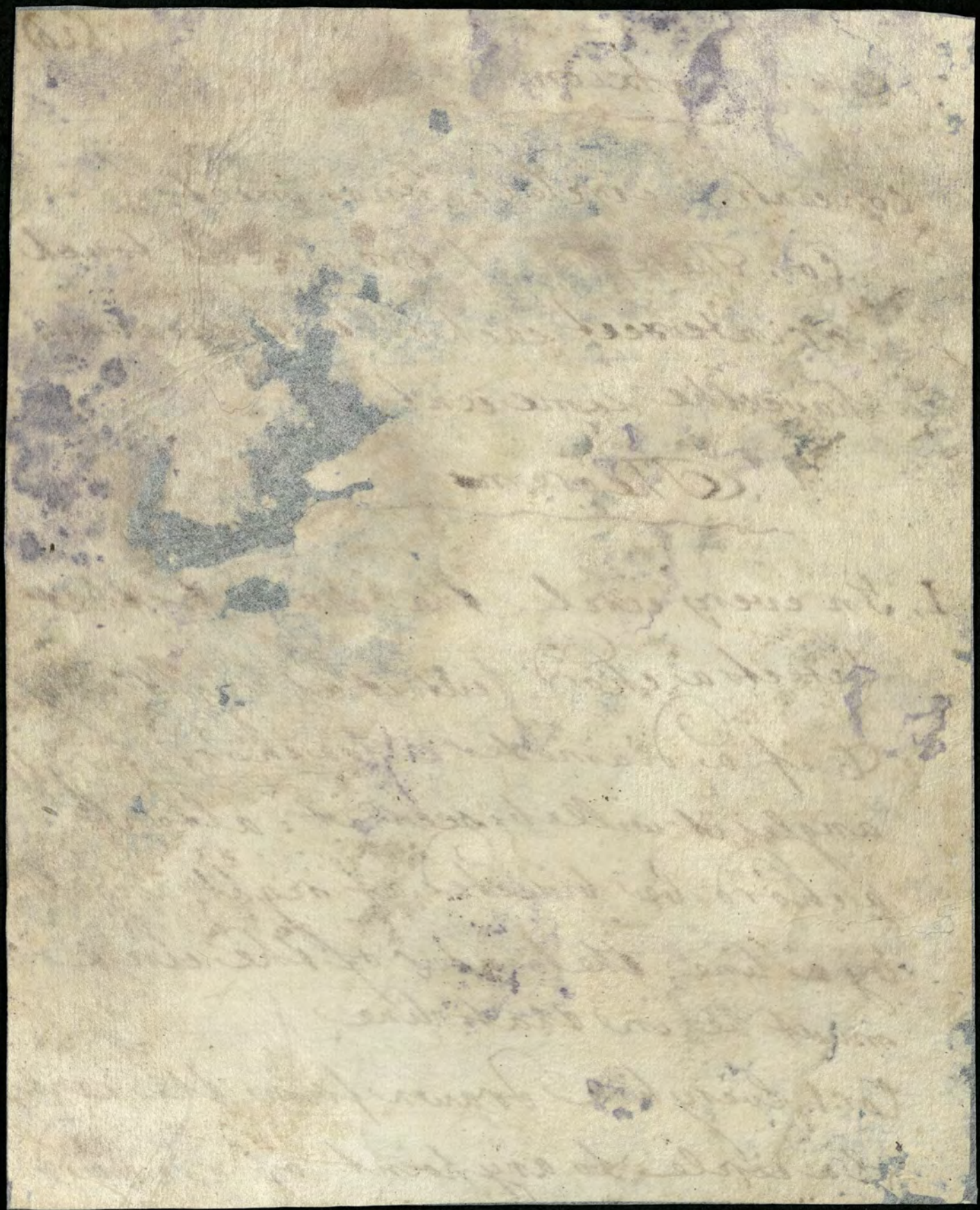
Concentric circles cannot meet.

Cor. Therefore if two circles touch or intersect each other they cannot have the same centre.

Theorems

I. In every circle the diameter that bisects a chord cuts it at right angles. & if a diameter cuts a chord at right angles, it will bisect it: also if a chord be bisected at right angles by a line, the center of the circle must be in that line.

Cor. 1. Every line drawn from the centre of a circle to any point of a chord

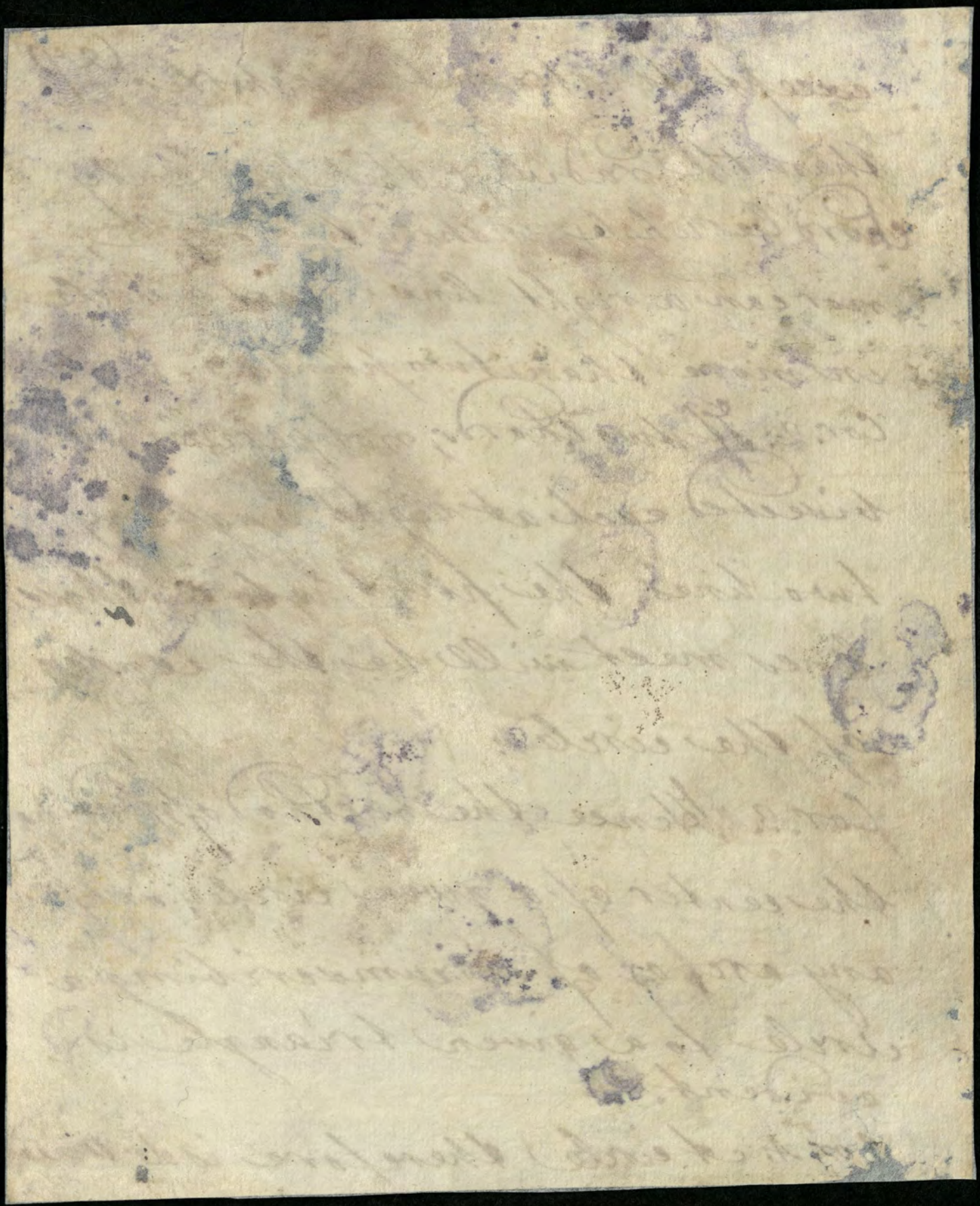


except its extremities is shorter than the radius; & therefor every chord lies wholy within the circle; nor can a right line cut a circle in more than two points.

Cor. 2. If two chords, not parallel, be bisected each at right angles by two lines, the point where those lines meet will be the center of the circle.

Cor. 3. Hence the method of finding the center of a given circle, or of any arc, or of circumscribing a circle to a given triangle is evident.

Cor. 4. A circle therefore is determin



(1852)

when three points in its circumference
are given; nore one " meet
another in mo. tha two points.

Cor. 5. The only point from whence
more than two equal lines can be
drawn to the circumf ence of
a circle is its center. dle.

Cor. 6. Two chords cannot bi ect
each other unless they pa thrugh e
center of the circle.

Cor. 7. Equal chords are equally distant
from the center of the circle: & conversely.

Theor. II.