Section III

Theor. II. The perpendicular at the extremity of a diameter is a tangent of the circle circumf. of Circle and every other line drawn thro' the point of contact

Cor. 1. Therefore if a right line be a tangent to a circle, it will be perpendicular to the diameter drawn from the point of contact.

Cor. 2. The perpendicular to a tangent at the point of contact passes through the center of the circle.

Cor. 3. If a right line touches two circles in the same point they will touch one another.

& if two circles touch, the tangent to the one at the point of contact will be a tangent to the other at the same point

Cor. 4. If two circles touch; the right line joyning their centers will pass through the point of contact.

III. The angle at the center is double of the angle at the circumference standing upon the same arc.

Cor. 1. The angle in a semi-circle is a right angle; the angle in a segment less than a semi-circle is obtuse; & the angle in a segment greater than a semi-circle is acute.

Cor. 2. All angles at the circumference standing upon the same arc are equal & therefore all angles in the same segment are equal.

Cor. 3. From Cor. 1. the method of drawing

a tangent to a given circle from a given point out of it, is evident.

IV. The sum of the opposite angles of a quadrilateral figure inscrib'd in a circle are equal to two right angles.

Lemma. Similar segments describ'd upon equal right lines, are equal.

V. In equal circles, equal angles, whether at the center or circumference, stand upon equal arcs: and conversely, the angles standing upon equal arcs, are equal.

VI. In equal circles if two chords be equal they subtend equal arcs: & conversly, if two arcs be equal they will be subtended by equal chords.

In like manner
~~Also,~~ the greater chord will subtend the greater arc: & the greater arc will be subtended by the greater chord.

Cor. 1. Hence it appears that the radius bisecting a chord, bisects its arc also: & that the radius bisecting the arc bisects the chord.

Cor. 2. The tangent at the middle of an arc is parallel to the chord.

Cor. 3. Hence the method of inscribing a square in a given circle, & of circumscribing a square to a given circle, is evident.

Cor. 4. From hence also appears the method of inscribing and circumscribing regular polygons, of 8, 16, 32. &c. sides.

VII. The angle form'd by the tangent of a circle & a chord drawn from the point of contact, is equal to the angle in the alternate segment.

Cor: 1. Hence the method of describing a segment upon a given right line, capable of a given angle is evident.

Cor: 2. As is also the method of cutting off from a given circle a segment capable of a given angle.

VIII. The side of a regular hexagon inscrib'd in a circle is equal to the radius

Cor: 1. Hence regular polygons of 12, 24, &c. sides, may be inscrib'd in & circum-scrib'd to a circle.

Cor: 2. The side of an equilateral triang

inscrib'd in a circle subtends an arc double to that subtended by the radius.

Section IV
Of the Proportions of lines & figures

Definition

Similar rectilinear figures are those which have their several angles equal each to each & the sides about the equal angles proportional.

Axiom

Geometrical magnitudes are to each other as the quantities expressing their respective measures according to any assum'd measure or unit.

Thus if a denote the quantity of the line X according to any assumed measure or unit, as a foot, inch &c. and if b denote the quantity of the line Y according to the same measure; then will $X : Y :: a : b$. or $\frac{X}{Y} = \frac{a}{b}$.
So if the quantity of the sides AB, BC, of the rectangle $ABCD$, be denoted respectively by a, b; & if the quantity of the sides EF, FG, of the rectangle $EFGH$, be denoted respectively, by c, d; the rectangles will be to each other, as ab, cd, that is $ABCD : EFGH :: ab : cd$. or $\frac{ABCD}{EFGH} = \frac{ab}{cd}$.

Hence it is usual to denote a rectangle $ABCD$, by the product of its sides $AB \times BC$, which signifies the as ab, that is the product

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of the quantity of AB into the quantity of BC, according to any common measure.

Lemma 1. If four lines be proportional, that is, if the first be to the second as the third is to the fourth, the rectangle under the extremes will be equal to the rectangle ~~of~~ under the means.

~~Lemma~~

Cor. If three lines be in continued proportion the square of the mean will be equal to the rectangle under the extremes.

Lemma 2. If there be any number of lines either in continued proportion or not, the ratio of the first to the last will be compounded of all the intermediate ratios; that is, of the ratio of the

first to the second, of the second to third, of
the third to the fourth, &c.

Cor. 1. Hence if three lines be in continued proportion the first will be to the third in the duplicate ratio of the first to the second, or as the square of the first is to the square of the second.

Cor. 2. If four lines be in continued proportion the first will be to the fourth in a triplicate ratio of the first to the second.

Theorems

I. Parallelograms of the same hight are to each other as their bases.

Cor. 1. Parallelograms of equal bases are to each other as their hights.

Cor. 2. Triangles of the same height are to each other as their bases, & triangles of equal bases are to each other as their heights.

II. If a line be drawn parallel to the base of a triangle it will cut its sides proportionally: & if the sides of a triangle be cut proportionally, the right line which joins the points of section will be parallel to the base.

Cor. 1. The line bisecting any angle of a triangle will cut the side opposite to that angle proportionally to the other sides.

Cor. 2. Equiangular triangles are similar.

Cor. 3. If two triangles have their sides mutually proportional the triangles will be equiangular.

Cor. 4. If a triangle have one angle equal to one angle of another triangle & if the (3) sides about the equal angles be proportional the triangles will be equiangular.

Cor. 5. Hence appears the method of cutting a given right line in the same proportion that another right line is ~~cutting~~ cut in: & also the method of dividing a right line into any given number of parts.

Cor. 6. Hence also the method of finding a third proportional to two given lines; & a fourth proportional to three given lines, appears.

II. The perpendicular drawn from the right angle to the hypothenuse of a rightangled triangle, divides it into two triangles similar to each other &

to the whole.

Cor. 1. Hence the perpendicular let fall from any point of the circumference of a circle to its diameter is a mea[n] proportional between the parts of the diameter.

Cor. 2. Hence a mean proportional between two given lines may be found.

IV. Equiangular parallelograms are to each other in a ratio compounded of the ratios of their sides.

Cor. If a triangle have one angle equal to one angle of another triangle the triangles will be to each other in the compound ratio of the sides comprehending the equal angle.

V. Similar triangles are to each other as the squares of their homologous sides,

or in the duplicate ratio of thos[e]

3. Hence all similar rectilinea[r]
are to each other as the squares of their
homologous sides.

VI. If two right lines intersect each other in a
circle the rectangle contain'd under the
segments of the one, will be equal to the
rectangle contain'd under the segments
of the other.

VII. If from any point without a circle a ~~tang~~ tangent
& a secant be drawn to it, the square of
the tangent will be equal to the rect=
angle under the whole secant, & its
external part.

Cor. 1. A tangent to a circle is a mean propor=
tional between a secant drawn from the
same point, & its external part.

Cor: 2. If two secants be drawn from the same point without the circle the rectangle under one secant & its external part will be equal to the rectangle under the other secant & its external part.

VIII. Angles at the center, or at the circumference of a circle are proportional to the arcs upon which they stand.

Cor. Hence the arc of a circle described between the sides of an angle with any radius, the center being in the angular point is the measure of the angle.

Finis