

1885

## Section III

(11)

Theor. II. The perpendicular at the extremity  
 of a diameter is a tangent of the circle  
 and every other line drawn thro' the point of contact

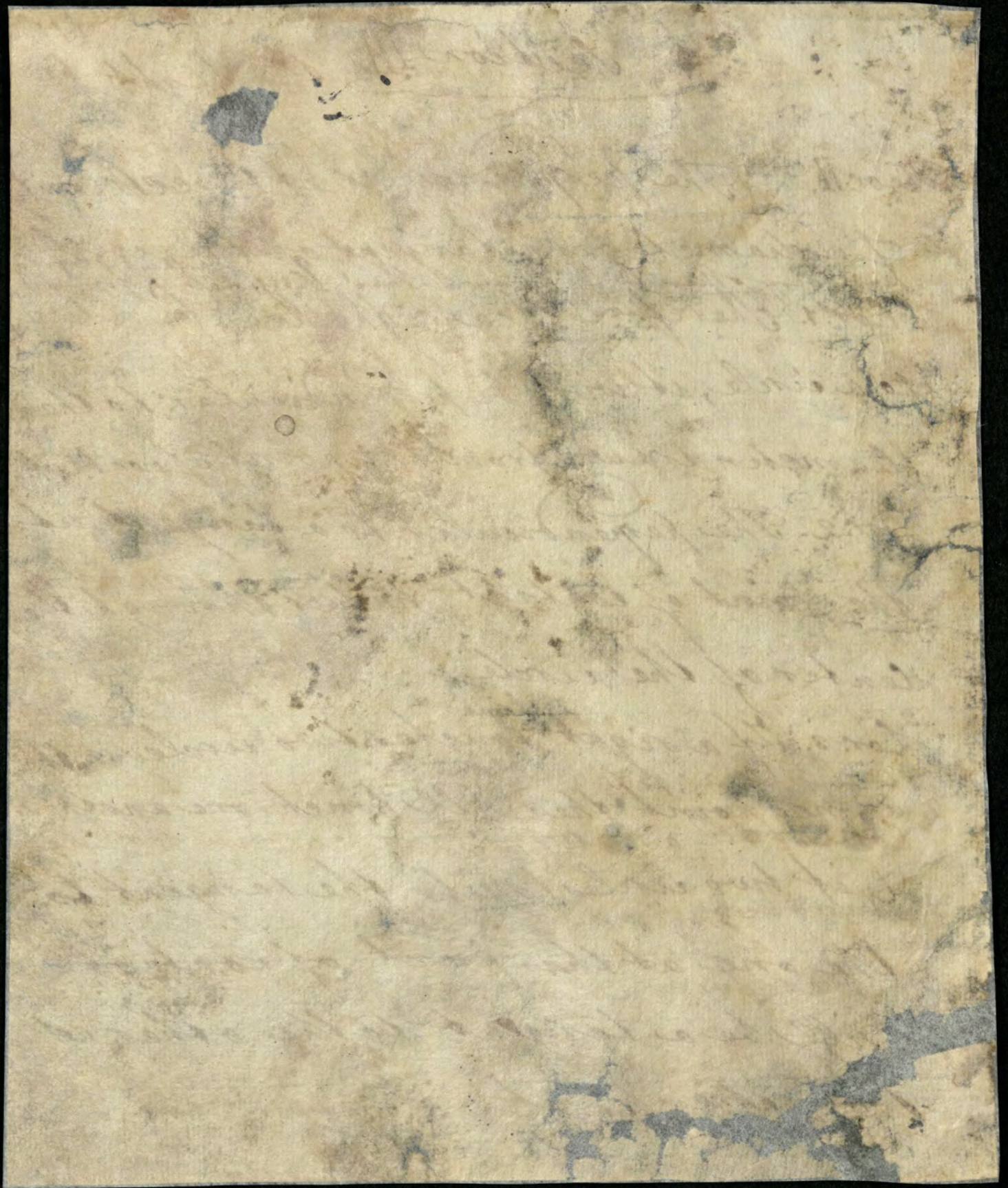
Cor. 1. Therefore if a right line be a tangent  
 to a circle, it will be perpendicular to the  
 diameter drawn from the point of contact.

Cor. 2. The perpendicular to a tangent at  
 the point of contact passes through the  
 center of the circle.

Cor. 3. If a right <sup>line</sup> touches two circles in the  
 same point they will touch one another.

& if two circles touch, the tangent to  
 the one at the point of contact  
 will be a tangent to the other at  
 the same point.







1886

Cor. 4. If two circles touch, the right line joining their centers will pass through the point of contact.

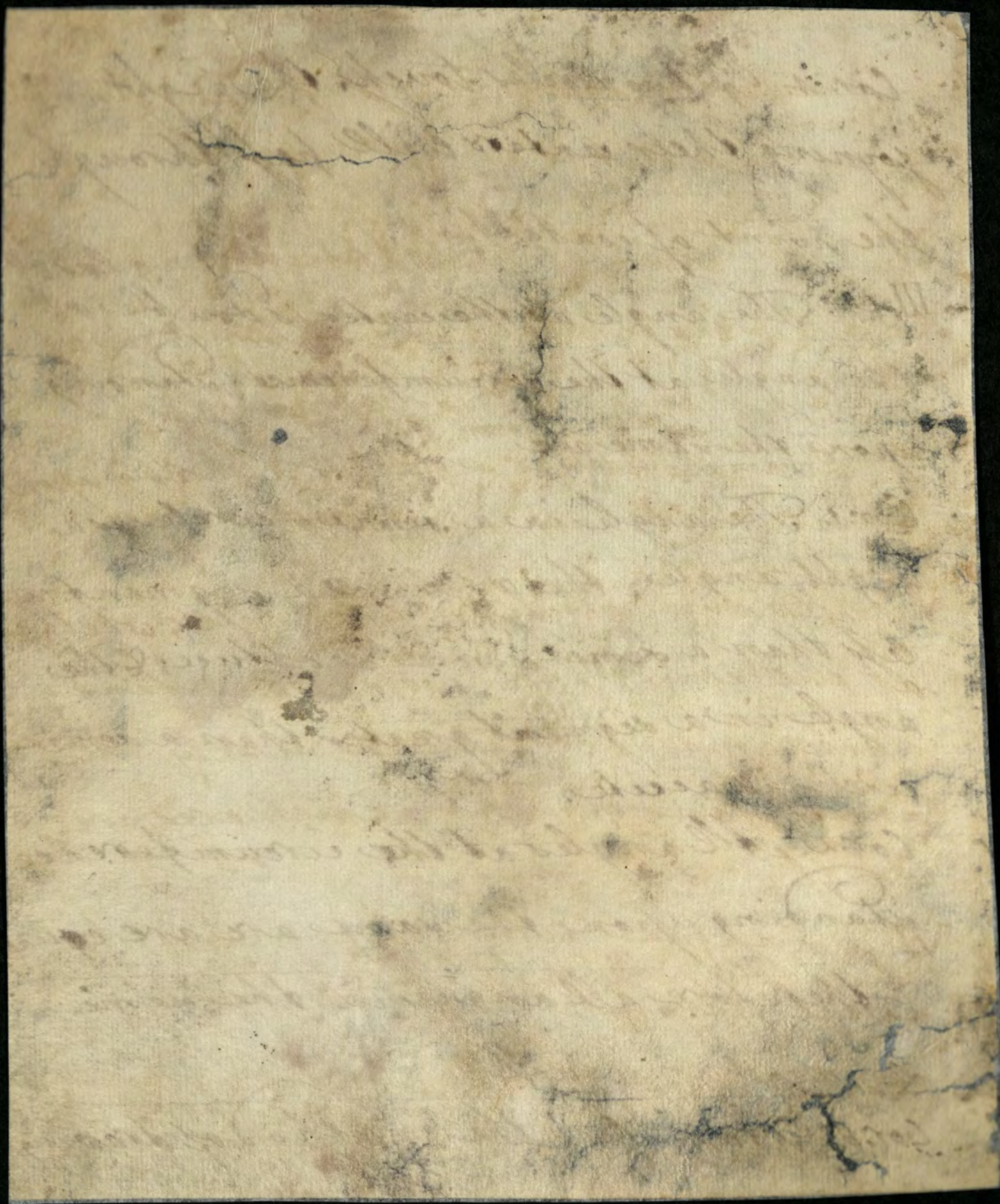
III. The angle at the center is double of the angle at the circumference standing upon the same arc.

Cor. 1. The angle in a semi-circle is a right angle; the angle in a segment less than a semi-circle is obtuse; & the angle in a segment greater than a semi-circle is acute.

Cor. 2. All angles at the circumference standing upon the same arc are equal; & therefore all angles in the same segment are equal.

Cor. 3. From Cor. 1. the method of drawing







1887

a tangent to a given circle from a given point out of it, is evident.

IV. The sum of the opposite angles of a quadrilateral figure inscribed in a circle are equal to two right angles.

Lemma. Similar segments describ'd upon equal right lines, are equal.

V. In equal circles, equal angles, whether at the center or in circumference, stand upon equal arcs: and conversely, the angles standing upon equal arcs, are equal.

VI. In equal circles if two chords be equal they subtend equal arcs: & conversely if two arcs be equal they will be subtended by equal chords.



*[Faint, illegible cursive handwriting on aged, stained paper]*



In like manner

Also, the greater chord will subtend the <sup>greater</sup> arc: & the greater arc will be subtended by the greater chord.

Cor. Hence it appears that the radius bisecting a chord, bisects its arc also: & that the radius bisecting the arc bisects the chord.

Cor. 2. The tangent at the middle of an arc is parallel to the chord.

Cor. 3. Hence the method of inscribing a square in a given circle, & of circumscribing a square to a given circle, is evident.

Cor. 4. From hence also appears the method of inscribing and circumscribing regular polygons, of 4, 6, 8, 12, 16, 24, 32. &c. sides.



*[Faint, illegible handwritten text, likely bleed-through from the reverse side of the page.]*



VII. The angle form'd by the tangent of a circle & a chord drawn from the point of contact, is equal to the angle in the alternate segment.

Cor. 1. Hence the method of describing a segment upon a given right line, capable of a given angle is evident.

Cor. 2. It is also the method of cutting off from a given circle a segment capable of a given angle.

VIII. The side of a regular hexagon inscrib'd in a circle is equal to the radius.

Cor. 1. Hence regular polygons of 12, 24,

48 &c. sides, may be inscrib'd & circumscrib'd to a circle.

Cor. 2. The side of an equilateral trian-



VII

VIII



inscrib'd in a circle subtends an arc  
double to that subtended by the  
radius.

Section IV  
Of the Proportions of  
lines & figures

Definition

Similar rectilinear figures are those  
which have their several angles equal  
each to each & the sides about the  
equal angles proportional.

Axiom

Geometrical magnitudes are to  
each other as the quantities expressing  
their respective measures according  
to any assum'd measure or unit.



*[Faint, illegible handwritten text in cursive script, likely a historical document or letter, showing signs of age and damage.]*



1891

Thus if  $a$  denote the quantity of the line  $X$  according to any assumed measure or unit, as a foot, inch &c. And if  $b$  denote the quantity of the line  $Y$  according to the same measure, then will  $X:Y::a:b$ , or  $\frac{X}{Y} = \frac{a}{b}$ .

So if the quantity of the sides  $AB, BC$ , of the rectangle  $ABCD$ , be denoted respectively by  $a, b$ ; & if the quantity of the sides  $EF, FG$ , of the rectangle  $EFGH$ , be denoted respectively, by  $c, d$ ; the rectangles will be to each other, as  $a, b, c, d$ , that is

$$ABCD: EFGH:: a, b: c, d. \text{ or } \frac{ABCD}{EFGH} = \frac{a, b}{c, d}.$$

Hence it is usual to denote a rectangle  $ABCD$ , by the product of its sides  $AB \times BC$ , which signifies the  $a, b$ , that is the product



... of a line the quantity of the  
... according to any given manner  
... or a foot, width &c. and of a breadth  
... of the line of accuracy to be  
... than with 2. 1/2. or 3. 1/2.  
... of the quantity of the width of the  
... be made to be as wide as  
... of the quantity of the width of the  
... of the width of the width of the  
... by a line of accuracy will  
... to each other in a line that is  
... of the width of the width of the  
... it is usual to connect a line  
... by the product of the width of the  
... in the width of the width of the



1892  
 of the quantity of  $AB$  into the quantity of  $BC$ , according to any common measure.

Lemma 1. If four lines be proportional, that is, if the first be to the second as the third is to the fourth, the rectangle under the extremes will be equal to the rectangle under the means.

~~Lemma 1.~~

Cor. If three lines be in continued proportion the square of the mean will be equal to the rectangle under the extremes.

Lemma 2. If there <sup>be</sup> any number of lines either in continued proportion or not, the ratio of the first to the last will be compounded of all the intermediate ratios; that is, of the ratio of the



*[Faint, illegible handwritten text, likely bleed-through from the reverse side of the page.]*



1893  
first to the second, of the second to the third,  
the third to the fourth, &c.

Cor. 1. Hence if three lines be in continued  
proportion the first will be to the third  
in the duplicate ratio of the first to the  
second, or as the square of the first is to  
the square of the second.

Cor. 2. If four lines be in continued propor-  
tion the first will be to the fourth in a  
triplicate ratio of the first to the second.

Theorems

I. Parallelograms of the same height are  
to each other as their bases.

cor. 1. Parallelograms of equal bases are  
to each other as their heights.



*[Faint, illegible cursive handwriting on aged, stained paper]*



1894  
 Cor. 2. Triangles of the same height are to each other as their bases, & triangles of equal bases are to each other as their heights.

II. If a line be drawn parallel to the base of a triangle it will cut its sides proportionally: & if the sides of a triangle be cut proportionally, the right line which joins the points of section will be parallel to the base.

Cor. 1. The line bisecting any angle of a triangle will cut the side opposite to that angle proportionally to the other sides.

Cor. 2. Equiangular triangles are similar.

Cor. 3. If two triangles have their sides mutually proportional the triangles will be equiangular.



12  
The first of the three  
The second of the three  
The third of the three  
The fourth of the three  
The fifth of the three  
The sixth of the three  
The seventh of the three  
The eighth of the three  
The ninth of the three  
The tenth of the three  
The eleventh of the three  
The twelfth of the three  
The thirteenth of the three  
The fourteenth of the three  
The fifteenth of the three  
The sixteenth of the three  
The seventeenth of the three  
The eighteenth of the three  
The nineteenth of the three  
The twentieth of the three  
The twenty-first of the three  
The twenty-second of the three  
The twenty-third of the three  
The twenty-fourth of the three  
The twenty-fifth of the three  
The twenty-sixth of the three  
The twenty-seventh of the three  
The twenty-eighth of the three  
The twenty-ninth of the three  
The thirtieth of the three  
The thirty-first of the three  
The thirty-second of the three  
The thirty-third of the three  
The thirty-fourth of the three  
The thirty-fifth of the three  
The thirty-sixth of the three  
The thirty-seventh of the three  
The thirty-eighth of the three  
The thirty-ninth of the three  
The fortieth of the three  
The forty-first of the three  
The forty-second of the three  
The forty-third of the three  
The forty-fourth of the three  
The forty-fifth of the three  
The forty-sixth of the three  
The forty-seventh of the three  
The forty-eighth of the three  
The forty-ninth of the three  
The fiftieth of the three  
The fifty-first of the three  
The fifty-second of the three  
The fifty-third of the three  
The fifty-fourth of the three  
The fifty-fifth of the three  
The fifty-sixth of the three  
The fifty-seventh of the three  
The fifty-eighth of the three  
The fifty-ninth of the three  
The sixtieth of the three  
The sixty-first of the three  
The sixty-second of the three  
The sixty-third of the three  
The sixty-fourth of the three  
The sixty-fifth of the three  
The sixty-sixth of the three  
The sixty-seventh of the three  
The sixty-eighth of the three  
The sixty-ninth of the three  
The seventieth of the three  
The seventy-first of the three  
The seventy-second of the three  
The seventy-third of the three  
The seventy-fourth of the three  
The seventy-fifth of the three  
The seventy-sixth of the three  
The seventy-seventh of the three  
The seventy-eighth of the three  
The seventy-ninth of the three  
The eightieth of the three  
The eighty-first of the three  
The eighty-second of the three  
The eighty-third of the three  
The eighty-fourth of the three  
The eighty-fifth of the three  
The eighty-sixth of the three  
The eighty-seventh of the three  
The eighty-eighth of the three  
The eighty-ninth of the three  
The ninetieth of the three  
The ninety-first of the three  
The ninety-second of the three  
The ninety-third of the three  
The ninety-fourth of the three  
The ninety-fifth of the three  
The ninety-sixth of the three  
The ninety-seventh of the three  
The ninety-eighth of the three  
The ninety-ninth of the three  
The hundredth of the three



1895  
 Cor. A. Triangles have one angle equal to one angle of another triangle & if the sides about the equal angles be proportioned the triangles will be equiangular.

Cor. S. Hence appears the method of cutting a given right line in the same proportion that another right line is <sup>cut in</sup> ~~cut in~~ as also the method of dividing a right line into any <sup>given</sup> number of parts.

Cor. B. Hence also the method of finding a third proportional to two given lines; & a fourth proportional to three given lines, appears.

I. The perpendicular drawn from the right angle to the hypotenuse of a right angled triangle, divides it into two triangles similar to each other &



*[Faint, illegible cursive handwriting on aged, stained paper]*



to the whole.

Cor. 1. Hence the perpendicular let fall from any point of the circumference of a circle to its diameter is a mean proportional between the part of the diameter.

Cor. 2. Hence a mean proportional between two given lines may be found.

IV. Equiangular parallelograms are to each other in a ratio compounded of the ratios of their sides.

Cor. If a triangle have one angle equal to one angle of another triangle the triangles will be to each other in the compound ratio of the sides comprehending the equal angle.

V. Similar triangles are to each other as the squares of their homologous sides,



*[The text on this page is extremely faint and illegible due to fading and damage. It appears to be a handwritten document, possibly a letter or a report, written in a cursive script. The paper is heavily stained and discolored, particularly at the top and bottom edges.]*



1897

or in the duplicate ratio of those  
 Hence all similar rectilinear  
 are to each other as the squares of their  
 homologous sides.

VI. If two right lines intersect each other in  $P$   
 circle the rectangle contain'd under the  
 segments of the one, will be equal to the  
 rectangle contain'd under the segments  
 of the other.

VII. If from any point without a circle a tangent  
 & a secant be drawn to it, the square of  
 the tangent will be equal to the rect-  
 angle under the whole secant, & its  
 external part.

Cor. 1. A tangent to a circle is a mean propor-  
 tional between a secant drawn from the  
 same point, & its external part.



*[Faint, illegible handwritten text, likely bleed-through from the reverse side of the page. The script is cursive and spans the entire page.]*



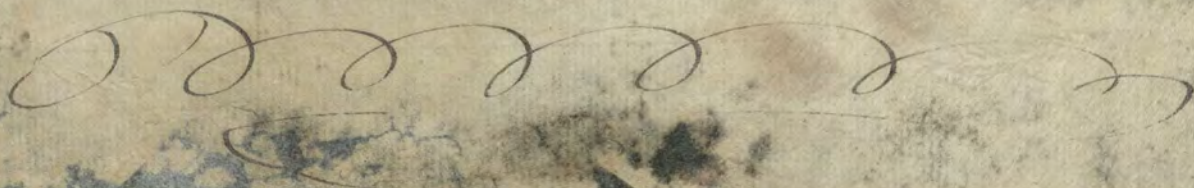
(1898)

Cor. 2. If two secants be drawn from the same point without the circle the rectangle under one secant & its external <sup>part</sup> will be equal to the rectangle under the other secant & its external part.

VIII. Angles at the center, or at the circumference of a circle are proportional to the arcs upon which they stand.

Cor. Hence the arc of a circle described between the sides of an angle with any radius, the center being in the angular point is the measure of the angle.

Finis





*[The page contains several lines of extremely faint, mirrored handwriting, likely bleed-through from the reverse side. The text is illegible due to fading and the quality of the scan. A large, faint signature or mark is visible in the lower right quadrant.]*